

6 FEBRUARY 2020

To obtain any credit, you must show your work! Place a box around each answer. (Note: Calculators are permitted, but not necessary.)

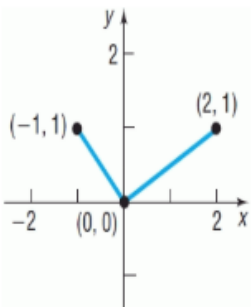
1. Consider the following piecewise-defined function:

$$G(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2020 & \text{if } 1 < x \leq 2 \\ \frac{1}{x} & \text{if } x > 2 \end{cases}$$

Find the exact value of each of the following:

- (a) $G(0) = 0$
- (b) $G(1) = 1$
- (c) $G(1.5) = 2020$
- (d) $G(2) = 2020$
- (e) $G(3) = 1/3$

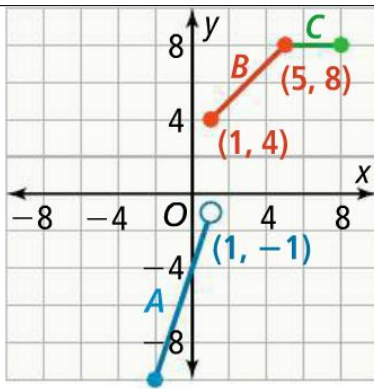
2. (a) Find the domain and range of the function graphed below:



Domain: $[-1, 2]$

Range: $[0, 1]$

(b) Find the domain and range of the function graphed below:



Domain: $[-2, 8]$

Range: $[-10, -1) \cup [4, 8]$

3. (a) Let $G(x) = \sqrt{x-1} + \sqrt[3]{x+44} + \frac{\sqrt{1+x^2}}{1+x^4}$. What is the domain of G ? Explain!


The domain consists of all real numbers x for which

$x - 1 \geq 0$. That is, $[1, \infty)$

(b) Find the domain of the function $g(x) = 2016 + \frac{x^2}{(x+3)(x-5)} + (x-1789)^4$

The domain consists of all real numbers except for $x = -3, x=5$.

4. In month $t = 0$, a small group of rabbits escapes from a ship onto an island where there are no rabbits. The island rabbit population, $p(t)$, in month t is given by



$$p(t) = \frac{1000}{1+(0.9)^t} \text{ for } t \geq 0.$$

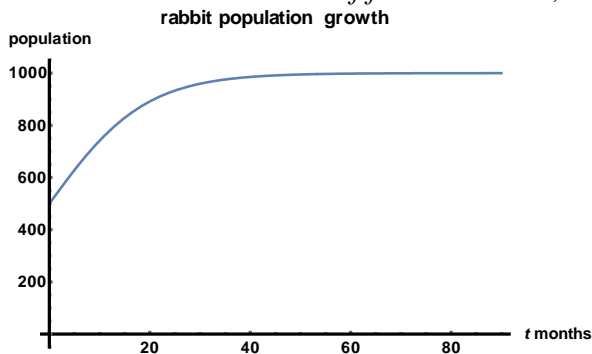
- (a) Evaluate $p(0)$, $p(10)$, $p(50)$, and explain their meaning in terms of rabbits.

- $p(0) = 500$, the initial population of rabbits
- $p(10) = 741$, the population after 10 months.
- $p(50) = 994$, the population after 50 months

In general, notice that as t grows larger and larger, $(0.9)^t$ grows closer and closer to 0.

- (b) Graph $p(t)$ for $0 \leq t \leq 100$. Describe the graph in words. Does it suggest the growth in the population you would expect among rabbits on an island?

Solution: At first, the growth of the rabbit population grows exponentially. Later, due to constraints on the island, such as a limited amount of food available, the population continues to increase, but very gradually.



- (c) Estimate the range of $p(t)$. What does this tell you about the rabbit population?

Answer: Range = $[500, 1000)$

EXTRA CREDIT:

Find the *domain* of the function

$$f(x) = 1732 + \sqrt{x-5} + \sqrt{8-x} + \frac{1+x+x^3}{x^2-18x+77}.$$

Express your answer either as one complete sentence or else in interval form.

Solution: Here, we need both $x - 5 \geq 0$ and $8 - x \geq 0$.

Also, since $x^2 - 18x + 77 = (x - 7)(x - 11)$, we must avoid $x = 7$ and $x = 11$.

Hence our answer is $[5,7) \cup (7,8]$.

The more you know, the less sure you are.
Voltaire