A spiral staircase in Nantes, France represents recursion — a sequence inside a sequence inside a sequence...

MATH 162: CLASS DISCUSSION

12 - 14 FEBRUARY 2020

NUMERICAL SERIES, *PART I*

I Review of sequences:

1) State the **Squeeze Theorem** for sequences.

2) Discuss the rules for convergence (and divergence) of a sum, difference, product, and quotient of two sequences. State the Squeeze (or Sandwich) Theorem for sequences. What is a *monotonic sequence*? What can be said about an increasing sequence that is bounded above? Is every bounded sequence convergent? Is every convergent sequence bounded? Give an example of two *divergent* numerical sequences whose sum is *convergent.* Explain why an *increasing bounded sequence* always converges. What is the corresponding result for decreasing sequences? Explain why the limit of a convergent sequence must be *unique*.

3) Determine *convergence* or *divergence*. Justify your answers.

(a) $\frac{(\ln\ln n)^{2525}}{n}$ (b) $c_n = \frac{\ln(n+2020\pi)}{\ln n}$ (c) $\frac{\cos\frac{5}{n}}{n}$ (d) $\int_0^n e^{-\pi t}\,dt$ (e) $\frac{(\ln\ln n)^{2525}}{n}$

(f) $\frac{100^n + 1789^n}{n! + 7^n}$ (g) $\sqrt{\frac{n+2020}{n} + e^{-\frac{3}{n}} + n\sin\frac{1}{n}}$ (h) $\left(1+\frac{1}{n}\right)^n$ (i) $\frac{3n^6+n+99}{(2n^2+1)^3}$.

(*j*) $\frac{(9n-888)}{\sqrt{81n^2+2525}}$

4) Consider the following recursive sequence: $a_1 = 2, a_2 = 4, a_{n+2} = \frac{a_n + a_{n+1}}{2}$ $for\ n \geq 1.$
Write the first five terms of this sequence.

5) Write recursive equations for the sequence 5,7, 9, 11,…

6) Write recursive equations for the sequence 2, 4, 8, 16, … 3.

7) What is the 5th term of the recursive sequence defined as follows:

$$a_1 = 5, a_n = 3a_{n-1}\ for\ n \geq 2.$$

8) What is the 7th term of the recursive sequence defined as follows:

$$a_1 = 2, a_n = 2a_{n-1}\ for\ n \geq 2.$$

II Introduction to numerical series

9) What is an infinite series? Explain precisely what it means for a series $\Sigma\, a_n$ to *converge*. What does it mean to say that a series *diverges*? What is meant by the *sum* of a series? In what sense is a series a particular type of sequence? Why is it important to distinguish between the *sequence of partial sums* and the *sequence of "atoms"*? What is a *positive series*?

10) Discuss the rules for convergence (and divergence) of a sum, difference, or constant multiple of a series.

11) What characterizes a *geometric series*? When is a geometric series convergent? How would one find the sum of a convergent geometric series?

12) For each of the following series, $\sum a_n$, determine *convergence* or *divergence*. If the series converges, are you able to find its sum?

(a) $\sum (-1)^n$ (b) $\sum 4^n / 3^n$ (c) $\sum 3 / 4^n$ (d) $\sum 9^n / 8^n$

13) Explain the unexpected behavior of the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$.

14) State and justify the *n^{th} term test for divergence*.

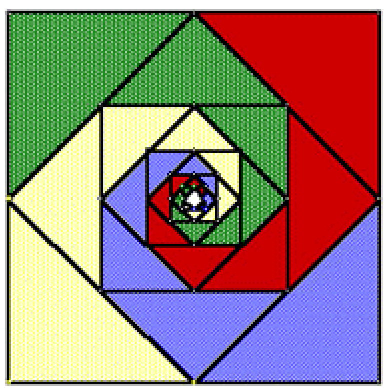
15) For each of the following series, $\sum a_n$, determine *convergence* or *divergence*. If the series converges, are you able to find its sum? (a) $\sum \arctan(k)$ (b) $\sum e^{-n}$ (c) $\sum (1/2^n + 1/3^n)$

(d) $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$ (e) $\sum_{j=1}^{\infty} \frac{(-1)^j}{13^j}$

(f) $\sum_{j=1}^{\infty} \frac{5^{j+1}}{3^{2j+3}}$ (g) $\sum_{n=1}^{\infty} \frac{1+n^{3.14}}{1+n^3}$ (h) $\sum_{n=1}^{\infty} \frac{\ln n}{\ln(\ln n)}$

16) State the *Comparison Test* for positive series.

17) How does this *Baravelle Spiral* represent an infinite series?



The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever. By using them, one may draw any conclusion he pleases, and that is why these series have produced so many fallacies and so many paradoxes.

- Niels Henrik Abel