MATH 162 class discussion 17 February 2020

Numerical Series continued *(revised)*

harmonic series; telescoping series; n^{th} term test; comparison test

1. Prove that the *harmonic series* diverges. (Divergence of the harmonic series was first demonstrated by Nicole d'Oresme (ca. 1323 –1382), but was mislaid for several centuries. Its name derives from the concept of overtones, or harmonics in music: the wavelengths of the overtones of a vibrating string are ½, 1/3, ¼, etc., of the string's fundamental wavelength. Every term of the series after the first is the harmonic mean of the neighboring terms; the phrase *harmonic mean* likewise derives from music.

2. Prove the *n^{th} term test for divergence*.
To which of the following series does the "*n^{th} term test for divergence*" apply? Explain!

$$(a)\ \sum_{n=1}^{\infty}\frac{n}{n+5}\quad (b)\ \sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n}\quad (c)\ \sum_{n=1}^{\infty}\frac{1}{n}\quad (d)\ \sum_{n=1}^{\infty}\frac{1}{n^3}$$

$$(e)\ \sum_{n=1}^{\infty}\arctan(n)\qquad (f)\ \sum_{n=1}^{\infty}n^{1/n}$$

3. For $n \ge 1$, let

$$a_n=\int_0^1 (x^2+2)^n\,dx.$$

Determine convergence or divergence of the *sequence* $\{a_n\}$. (*Hint:* Do *not* try to evaluate the integral! Calculator solutions are not accepted. Is the sequence *monotone, that is, strictly increasing or strictly decreasing*?)

4. Review *geometric* series. Find the exact value of 0.123123123…

5. Carefully state the **Comparison Test** for positive series.

6. For each of the following infinite series, determine *convergence* or *divergence. In the case of convergence, find the sum of the series:*

$$(a)\ \sum_{n=1}^{\infty}\ln\frac{n+1}{n}\quad (b)\ \sum_{n=0}^{\infty}\frac{5}{9^n}\quad (c)\ \sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n}\quad (d)\ \sum_{n=1}^{\infty}\frac{1}{n(n+1)}$$

(Hint: Calculate the first few partial sums.)

$$(e)\ \sum_{n=1}^{\infty}\cos\left(\frac{5}{n}\right)\qquad (f)\ 0.123412341234\ldots$$

7. (a) Use the comparison test to show that $\sum_1^\infty \frac{1}{n^2}$ converges.
Hint: Compare to the telescoping series $\sum_2^\infty \frac{1}{n(n-1)}$.

(b) What can you say about $\sum_1^\infty \frac{1}{n^p}$ where $p \ge 2$? *Hint: Compare* to $\sum_1^\infty \frac{1}{n^2}$.

(c) What can you say about $\sum_{n=1}^{\infty}\frac{1}{n^p}$ where $p \le 1$? *Hint:* Compare to $\sum_1^\infty \frac{1}{n}$

8. Find the *sum* of each of the following convergent series. Show your work.

$$(a)\ \sum_{n=0}^{\infty}\left(e^{-n}-e^{-n-1}\right)\quad (b)\ \sum_{k=0}^{\infty}\frac{(-4)^{k+1}}{5^{k-1}}\quad (c)\ 5.314314314314314\ldots$$

9. *(University of Michigan)* Consider the series $\sum_0^\infty \frac{9^n}{8^n+10^n}$.

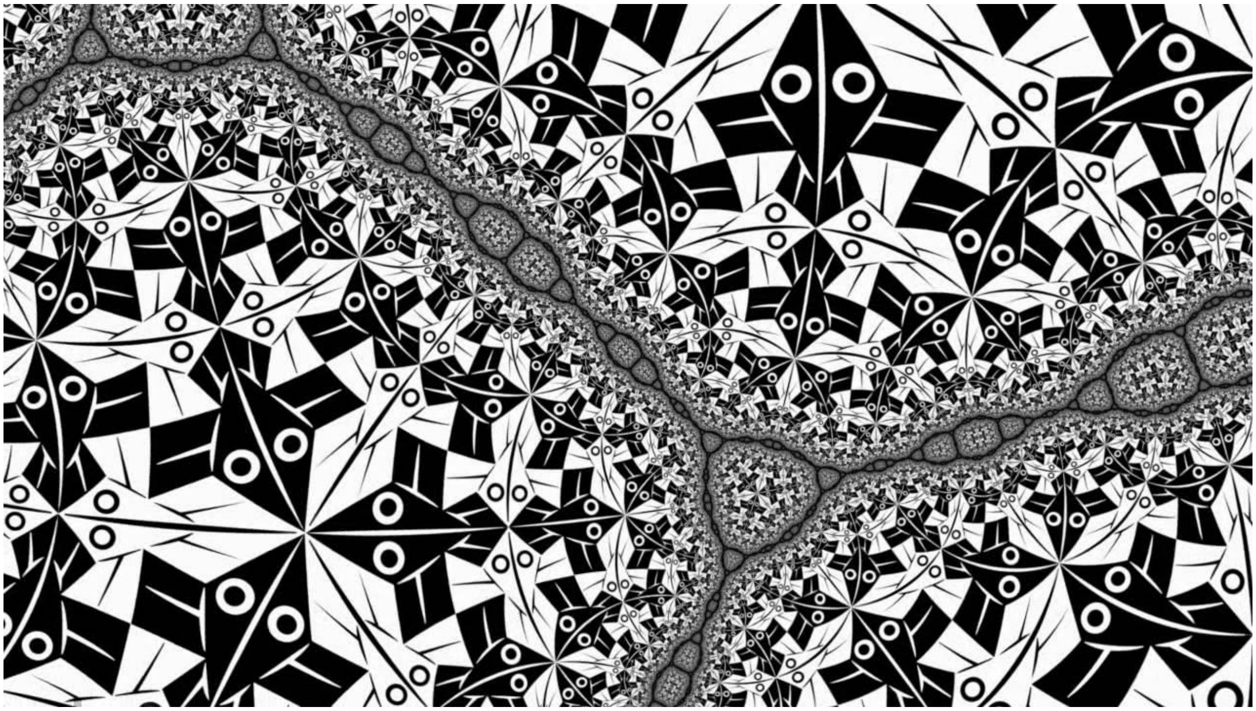
Use an appropriate series test to show that the series converges. Be sure to indicate which test(s) you are using. Also verify all hypotheses needed for the test, and justify the convergence/divergence of any other series you use.

10. *(University of Michigan)* Does the following series converge or diverge? Justif

$$\sum_{n=4}^{\infty} \frac{1}{n^3 + n^2 \cos(n)}$$

11. *(University of Michigan) Determine the convergence or divergence of the following series. Justify your answer.*

$$\sum_{n=1}^{\infty} \frac{9n}{e^{-n} + n}\ .$$



M. C. Escher: Circle Limit

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)