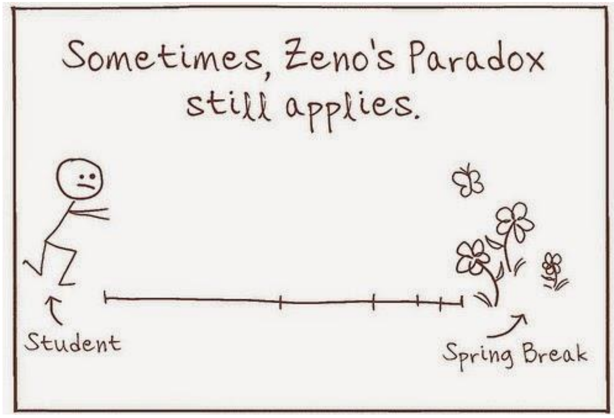
MATH 162 CLASS DISCUSSION 19 FEBRUARY 2020

MORE ON POSITIVE SERIES

(INTEGRAL TEST, RATIO TEST, ROOT TEST, LIMIT COMPARISON TEST)



1. For each of the following series, $\sum a_n$, determine *convergence* or *divergence*. Justify each answer.

$$(a)\ \sum_{n=1}^{\infty}\frac{1}{n(n+1)} \qquad (b)\ \sum_{n=1}^{\infty}\ln\frac{n}{n+1} \qquad (c)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt[3]{n}+1}$$

$(c)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt[3]{n}+1}$ $\quad (d)\ \sum_{n=1}^{\infty}\frac{n+3^n}{n^2 3^n}$ $\quad (e)\ \sum_{n=1}^{\infty}\sin\frac{1}{n}$ *Hint:* First, explain why, for small positive *x*, sin x > x/2.

$$(f)\ \sum_{n=1}^{\infty}\frac{1}{1+\ln n} \qquad (g)\ \sum_{n=4}^{\infty}\frac{1}{n(\ln n)(\ln\ln n)^2} \qquad (h)\ \sum_{n=1}^{\infty}\frac{1}{n^n}$$

$$(i)\ \sum_{n=1}^{\infty}\frac{1}{n^{\ln n}} \qquad (j)\ \sum_{n=1}^{\infty}\frac{1}{n^{1/n}} \qquad (k)\ \sum_{n=1}^{\infty}\frac{5^n(n!)^2}{(2n)!} \qquad (l)\ \sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$$

$$(m)\ \sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2} \qquad (n)\ \sum_{n=1}^{\infty}\frac{2(4)(6)\ldots(2n)}{1(3)(5)\ldots(2n-1)} \qquad (o)\ \sum_{n=1}^{\infty}\frac{\arctan n}{1+n^2}$$

$$(p)\ \sum_{n=1}^{\infty}\frac{\ln(n^5+n^3+11)}{n^2} \qquad (q)\ \sum_{n=1}^{\infty}\frac{1+2^n+9^n}{5^n+8^n+n^{13}} \qquad (r)\ \sum_{n=1}^{\infty}\frac{n^{2n}}{(1+2n^2)^n}$$

$$(s)\ \sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{n} \qquad (t)\ \sum_{n=1}^{\infty}\frac{\pi^n}{3^{2n}(2n)!} \qquad (u)\ \sum_{n=1}^{\infty}\{\arctan(n+1)-\arctan n\}$$

$$(v)\ \sum_{n=1}^{\infty}\frac{n!}{n^n} \qquad (w)\ \sum_{n=1}^{\infty}\frac{2^{n^2}}{n!}$$

2. (a) Explain why the *sequence* $n^{1/n}$ converges. What is its limit?
   (b) Show that the following *series* converges. (*Hint:* Use the root test.)

$$\sum_{n=2}^{\infty}\frac{n}{(\ln n)^n}$$

   (c) Using the fact that the series in (b) converges, explain why n = $o((\ln n)^n)$.
   (d) Determine if the following series converges or diverges. (*Hint:* Use the Comparison Test.)

$$\sum_{n=2}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$$

3. *(University of Michigan)* Suppose a_n and b_n are sequences of positive numbers with the following properties.

- $\sum_{n=1}^{\infty} a_n$ converges.
- $\sum_{n=1}^{\infty} b_n$ diverges.
- $0 < b_n \le M$ for some positive number M.

For each of the following questions, circle the correct answer. No justification is necessary.

a. [2 points] Does the series $\sum_{n=1}^{\infty} a_n b_n$ converge?

Converge Diverge Cannot determine

b. [2 points] Does the series $\sum_{n=1}^{\infty} (-1)^n b_n$ converge?

Converge Diverge Cannot determine

c. [2 points] Does the series $\sum_{n=1}^{\infty} \sqrt{b_n}$ converge?

Converge Diverge Cannot determine

d. [2 points] Does the series $\sum_{n=1}^{\infty} \sin(a_n)$ converge?

Converge Diverge Cannot determine

e. [2 points] Does the series $\sum_{n=1}^{\infty} (a_n + b_n)^2$ converge?

Converge Diverge Cannot determine

f. [2 points] Does the series $\sum_{n=1}^{\infty} e^{-b_n}$ converge?

Converge Diverge Cannot determine

4. *(University of Michigan)* rigorously determine whether or not the following series converge or diverge.

a. $\sum_{n=6}^{\infty} \frac{\ln(n)+3}{n-4}$ b. $\sum_{n=1}^{\infty} \frac{n+\sin(n)+1}{e^n - n - 1}$

5. State Stirling's formula. Using Stirling's formula write 100! in scientific notation.

With the exception of the geometrical series, there does not exist in all of mathematics a single infinite series the sum of which has been rigorously determined. In other words, the things which are the most important in mathematics are also those which have the least foundation.

- Niels Henrik Abel (1802 – 1829)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