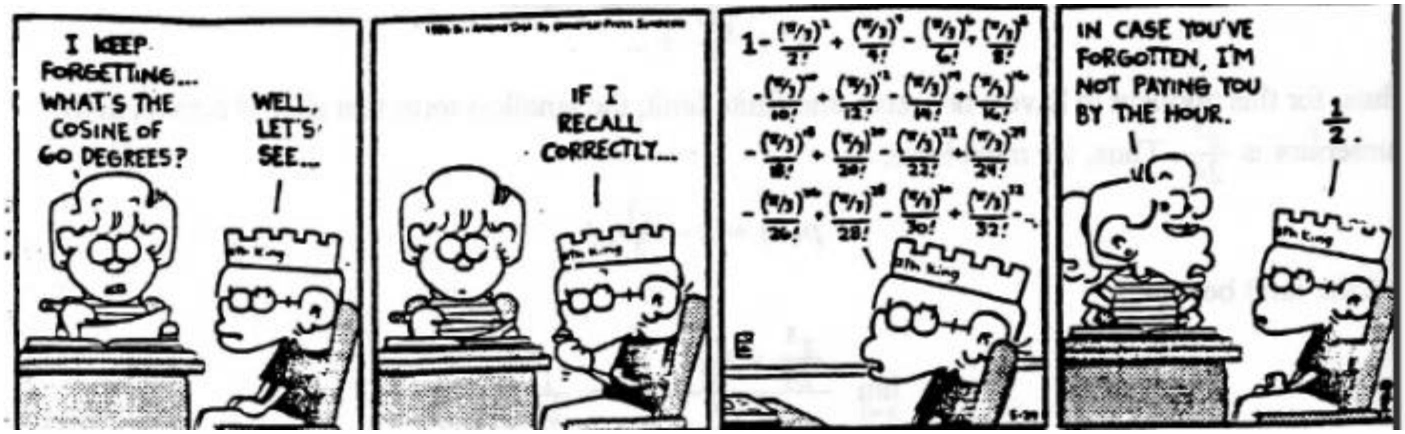
MATH 162 DISCUSSION: 24 FEBRUARY 2020

Absolute & conditional convergence



1. Explain how the ***ratio*** and ***root tests*** can be extended for series more general than positive series.

2. State the **Cauchy-Leibniz rule** for alternating series.

3. For each of the following series, determine *absolute* convergence, *conditional* convergence, or *divergence*.

$(a)\ \sum \frac{(-1)^n}{n^3}$ $(b)\ \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ $(c)\ \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$ $(d)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)}$

$(e)\ \sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{n^8}$ $(f)\ \sum_{n=1}^{\infty} \frac{(-1)^n(3n+5)}{2018n+1}$ $(g)\ \sum_{n=1}^{\infty} \frac{(-1)^n e^{-n}}{\sqrt{n+1}}$

$(h)\ \sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{\frac{3}{2}}}$ $(i)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$ $(j)\ \sum_{n=1}^{\infty} \frac{(-2)^{n+1}(3n+5)}{n+5^n}$

$(k)\ \sum_{n=1}^{\infty} (-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$ $(l)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$

4. *(University of Michigan final exam problem)*

a. Determine whether the following series converge or diverge (circle your answer). For each, justify your answer by writing what convergence rule or convergence test you would use to prove your answer. If you use the comparison test or limit comparison test, also write an appropriate comparison function.

(i)
$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2}$$
Converge Diverge

(ii)
$$\sum_{n=1}^{\infty} \frac{3n-2}{\sqrt{n^5+n^2}}$$
Converge Diverge

b. Does the following series converge conditionally, absolutely, diverge, or is it not possible to decide? Justify.
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(1+\ln(n))}$$

5. *(University of Michigan final exam problem)*
Determine whether the following series converge or diverge (circle your answer). Be sure to mention which tests you used to justify your answer. If you use the comparison test or limit comparison test, write an appropriate comparison function.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$ (b) $\sum_{n=1}^{\infty} ne^{-n^2}$ (c) $\sum_{n=1}^{\infty} \frac{\cos(n^2)}{n^2}$

Additional exercises:

For each numerical series below, determine *convergence* or *divergence*. In the case of convergence, determine if the series converges *absolutely* or *conditionally*. Justify each answer.

$(a)\ \sum_{m=1}^{\infty}\left(\frac{-e}{m}\right)^m$ $(b)\ \sum_{n=1}^{\infty}\frac{(-1)^{3n}2^n}{\left(1+\frac{1}{n}\right)^{n^2}}$ $(c)\ \sum_{n=3}^{\infty}(-1)^n\frac{5}{\ln n}$

$(d)\ \sum_{n=1}^{\infty}\frac{(-1)^n(n!)^2 3^n}{(2n+1)!}$ $(e)\ \sum_{k=1}^{\infty}(-1)^k\frac{k(k+1)(k^2+5)}{(k-13\ln k)^4}$

$(f)\ \sum_{n=1}^{\infty}\frac{5^n+7}{11^n}$ $(g)\ \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3n)!}{(n!)^3}$ $(h)\ \sum_{n=1}^{\infty}(-1)^n\frac{4^n(n!)^2}{(2n)!}$

2. For each of the following numerical series, determine if the series ***diverges***, ***converges conditionally***, **or** ***converges absolutely***. Justify your answers!

(a) $\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{1}{5+n^7\ln n}\right)$ (b) $\sum_{n=2}^{\infty}\frac{\sin(3n+5)}{n(\ln n)^2}$

(c) $\sum_{n=3}^{\infty}(-1)^n\frac{5}{\ln n}$ (d) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{1+n}{5+n^2}\right)^n$

(e) $\sum_{k=1}^{\infty}(-1)^k\frac{k(k+1)(k^2+5)}{(k-13\ln k)^4}$ (f) $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{\left(1+\frac{1.3}{n}\right)^{n^2}}$

3. Does absolute convergence imply convergence? Does convergence imply absolute convergence? Why?
4. How many terms are required to estimate each of the following sums accurately to 4 decimal places?

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^8}$ (c) $\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln n}$

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction.

– Leo Tolstoy