DISCUSSION: POWER SERIES

28 FEBRUARY & 9 MARCH 2020
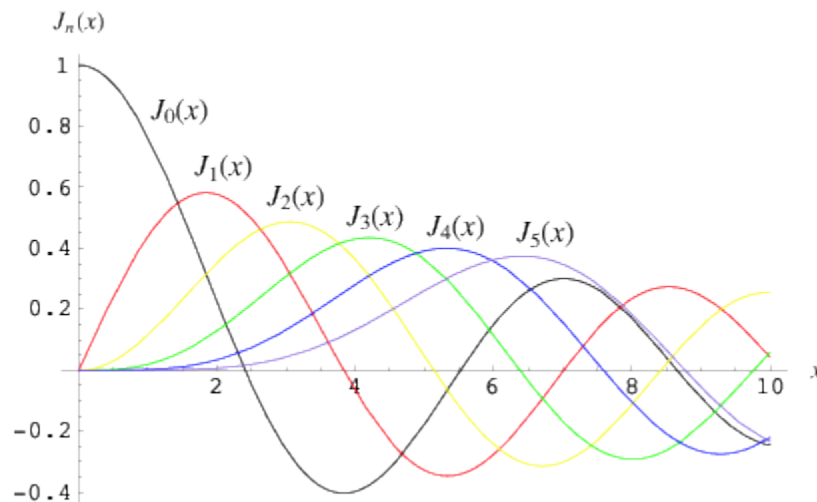
Bessel functions may be expressed as power series

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1. For each of the following power series, determine the *radius of convergence* and the *interval of convergence*. Consider end-point behavior as well.

(a) $\displaystyle\sum \frac{x^n}{n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^n}{(2n+1)}$

(e) $\displaystyle\sum_{n=1}^{\infty} n!\, x^n$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+5}{2016n+1}(x-1)^n$

(g) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n (x+4)^n$

(h) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-1)^n}{n\ln n}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{5^n}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n+5^n}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{n x^{2n}}{1+3^n}$

2. *[University of Michigan Final Exam question]* Consider the following power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n(2n+1)}(x-3)^n$$

(a) For which values of x does the power series converge?

(b) For which values of x does the power series converge absolutely?

(c) For which values of x does the power series converge conditionally?

3. *[University of Michigan Final Exam question]* Consider the following power series

$$\sum_{n=1}^{\infty} \frac{1}{4^n n^3}(x-2)^n$$

Find the interval of convergence of the power series. Justify your answer.

4. *[University of Michigan Final Exam question]* Consider the following power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(n+1)}(x+1)^{2n}$$

(a) At $x = -3$, does the series converge absolutely, conditionally, or diverge?

(b) Using just your answer in (a), state the *possible* values for the radius of convergence R. Justify.

(c) Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(n+1)}(x+1)^{2n}$$

5. Determine the *interval of convergence* of the following power series. (You need not study end-point behavior.)

$$\sum_{n=1}^{\infty} \frac{n^{13} 13^n}{\sqrt{n+2020}}(x-13)^n$$

6. For each of the following power series, find the interval of convergence and the radius of convergence.

(a) $\sum_{n=1}^{\infty}(-1)^n n^2 x^n$
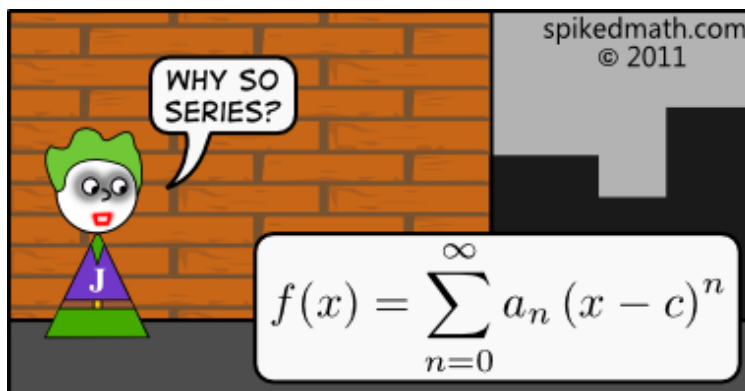
(b) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}(x-3)^n$

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{10^n}{n!}(x-10)^n$

(c) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}(x+1)^n$

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n 10^n}(x-2)^n$

Next time, we will view Taylor series examples on several videos, including
https://www.youtube.com/watch?v=3d6DsjIBzJ4

Pure mathematics is, in its way, the poetry of logical ideas.

- Albert Einstein

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

- John von Neumann