MATH 162 ADDITIONAL PRACTICE PROBLEMS

5 February 2020

1. Use integration by parts to evaluate the following integral:
$\int (\ln x)^2 \, dx$

2. Using integration by parts, evaluate

$$\int x \sec^2 x \, dx.$$

3. Using *integration by parts*, find an anti-derivative of each if the following functions:

(a) xe^x (b) $\sin(\ln x)$ (c) arc sin x

4. Determine *convergence* or *divergence* of each of the following improper integrals:

$$(a) \int_{0+}^{\frac{1}{e}} \frac{(-\ln x)}{x^4} dx \quad (b) \int_{0+}^{\infty} \frac{1}{\sqrt{x+x^4}} dx$$

$$(c) \int_{0+}^{\infty} \frac{5x+7}{\sqrt{x^2+3x^4}} dx$$

5. *Evaluate* each of the following convergent improper integrals. Show your work!

(a) $\displaystyle\int_0^\infty t^3 e^{-t^4} dt$ (b) $\displaystyle\int_3^\infty \frac{1}{x(1+\ln x)^{7/3}} dt$

6. For each of the following improper integrals, determine convergence or divergence. *Justify each answer! (That is, if you use the comparison test, exhibit the function that you choose to use for comparison and show why the appropriate inequality holds. Of course, you may also use the asymptotic comparison test.)*

(a) $\displaystyle\int_0^\infty \frac{1+x+x^4}{(1+x)^5} dx$ (b) $\displaystyle\int_0^\infty \frac{1+x+e^x}{5+3e^{3x}} dx$

7. For each of the following improper integrals, determine convergence or divergence.

$$\int_1^\infty \frac{2+\sin x}{\sqrt{x+1}} dx \qquad \int_1^\infty \frac{\theta}{\sqrt{\theta^5+1}} d\theta \qquad \int_0^1 \ln(x)\, dx \qquad \int_2^\infty \frac{x+\sin x}{x^2-x} dx$$

8. A machine produces copper wire, and occasionally there is a flaw at some point along the wire. The length X of the wire produced between two consecutive flaws is a continuous random variable with probability density function:

$$f(x) = \begin{cases} c(1+x)^{-3} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Show your work in order to receive full credit:

(a) Find the value of *c*.

(b) Find the *cumulative distribution function* P(x) of the density function f(x). Remember that a cumulative distribution function is defined on the entire real line.

(c) Find the *expected value* of the length of wire between two consecutive flaws.

9. *(Univ Michigan)* Albertine likes to juggle. So does Jean-Luc. The number of minutes Albertine can juggle five balls without dropping one is a random variable, with probability density function f(t) = 0.8e −0.8t . Similarly, the function j(t) = 1.5e −1.5t describes Jean-Luc's skill. Here t is time in minutes. a. (2 pts) Find R ∞ 0 f(t)dt. No need to show work. b. (5 pts) What percentage of Jean-Luc's juggling attempts are "embarrassing," meaning they last for 10 seconds or less? Show your work. c. (6 pts) How long can Albertine juggle, on average? Show your work. d. (5 pts) Who is the better juggler? Give a good reason for your decision.

"Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?" "I don't know," said Alice. "I lost count."

- Lewis Carroll, **Through the Looking Glass**