MATH 162: CLASS DISCUSSION

13 January 2020

INTEGRATION BY PARTS

1. Using *integration by parts*, find the indefinite integral of each of the following functions.

(a) $x \sin x$	(b) $x^2 \sin x$	(c) $x \ln x$
(d) $\ln x$	(e) $(\ln x)^2$	(f) $x e^{3x}$
(g) $\arcsin(2x)$	(h) $x\ arc\tan x$	(i) $\sec^3 x$

2. Evaluate: $\int e^x \cos x\, dx$. What makes this integral "unusual"?

(a) $\int e^{3x} \cos 4x\, dx$	(b) $\int \ln\sqrt{x}\ dx$	(c) $\int (arc\sin x)^2\ dx$
(d) $\int_1^2 t^2 \ln t\, dt$	(e) $\int_0^{\frac{1}{2}} x\cos(\pi x) dx$	(f) $\int \frac{x^3}{\sqrt{1+x^3}}\ dx$

3. Using an appropriate substitution followed by integration by parts, find an anti-derivative for each of the following.

(a) $e^{\sqrt{x}}$	(b) $e^{\cos x}\sin(2x)$	(c) $x\ln(1+x)$

4. Using an appropriate substitution followed by integration by parts, find the value of each of the following Riemann integrals.

(a) $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \alpha^3 cos^2\alpha\, d\alpha$	(b) $\int_0^{\pi} e^{\cos w}\sin(2w) dw$	(c) $\int_1^2 (\ln x)^2\, dx$

5. (a) Prove the reduction formula:

$$\int x^n \sin x\ dx = -x^n \cos x + n\int x^{n-1}\cos x\ dx$$

   (b) Using this reduction formula, compute an anti-derivative of $x^3 \sin x$.

6. (a) Verify the reduction formula:

$$\int \sec^n x\, dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$$

(b) Using this reduction formula, compute anti-derivatives of $\sec^4 x$ and $\sec^5 x$.

7. Solve each of the following differential equations.

$\frac{dy}{dt} = \frac{\ln x}{x^2}$	$\frac{dy}{dx} = xe^{-x}$	$\frac{dy}{dx} = sec^5 x$ Hint: $sec^5 x = sec^2 x sec^3 x$

Common integration is only the memory of differentiation.

- Augustus de Morgan (1806 – 1871)

Nature laughs at the difficulties of integration.

- Pierre-Simon de Laplace (1749 - 1827)