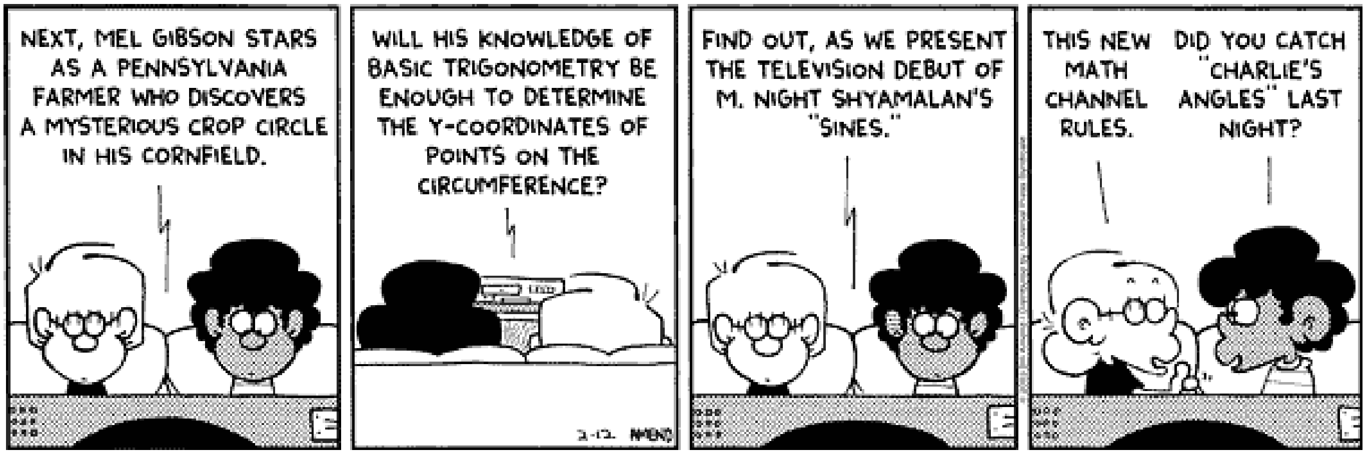
MATH 162 CLASS DISCUSSION 17 JANUARY 2020

TRIG SUBSTITUTION



Review

1) Solve the *differential equations*

(a) $\frac{dy}{dx} = tan^3x\, sec^2x + \tan x$ (b) $\frac{dy}{dx} = sin^3x\, cos^2x + \sin x \cos x$

2) Find the area between the curves

(a) $y = \sin^2 x$ and $y = \sin^3 x$ over the interval $[0, \pi]$.

(b) $y = \tan x$ and $y = \tan^2 x$ over the interval $[0, \pi/4]$.

3) Find the *average value* of the function

$(a)\quad f(x) = sin^2\,x\;\; cos^3x\;\; over\ the\ interval\ [-\pi, \pi].$

$(b)\quad f(x) = tan^2\,x\;\; over\ the\ interval\ \left[0, \frac{\pi}{4}\right].$

$(c)\quad f(x) = tan^4\,x\;\; over\ the\ interval\ [0, \pi/4]$

4) A particle travels on a straight line with velocity function $v(t) = \sin(\omega t)\, cos^2\,(\omega t)$. Find its position function s(t) if f(0) = 0. (This is called an initial-value problem.)

5) Find the indefinite integral of

(a) $sin^2\,x\; cos^2\,x$ $(b)\quad sec^9\,x\; tan^5\,x$ $(c)\quad sec^4\,x\; tan^6\,x$

6) **Challenge problem** (*U. Michigan, exam 1, Oct 2016*)

Suppose that *f* is a twice-differentiable function that satisfies

f(0) = 1, f(2) = 2, f(4) = 4, $f'(2) = 3$, $\int_0^2 f(x)\,dx = 5$, $\int_2^4 f(x)\,dx = 7$

Evaluate each of the following integrals:

(a) $\int_0^2 xf'(x)\,dx$ (b) $\int_{\sqrt{2}}^2 xf'(x^2)\,dx$ (c) $\int_0^2 x^3 f'(x^2)\,dx$

Solutions:

http://www.math.lsa.umich.edu/courses/116/Exams/2016_17fall16/Exam1/exam1_Solutions_F16.pdf

TRIG SUBSTITUTIONS By making an appropriate trig (or hyperbolic) substitution, convert each of the following integrals to trig integrals. Do not evaluate.

I. (a) $\int \frac{x}{(x^2+1)^3}\,dx$ (b) $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$ (c) $\int \frac{x^2}{\sqrt{x^2-1}}\,dx$ (d) $\int \frac{\sqrt{1-x^2}}{x^2}\,dx$

II.

Exercises from UC Davis. **SOLUTIONS** at
https://www.math.ucdavis.edu/~kouba/CalcTwoDIRECTORY/trigsubdirectory/TrigSub.html

- PROBLEM 1 : Integrate $\int \sqrt{1-x^2}\,dx$
- PROBLEM 2 : Integrate $\int \frac{(x^2-1)^{3/2}}{x}\,dx$.
- PROBLEM 3 : Integrate $\int \frac{1}{(1-x^2)^{3/2}}\,dx$.
- PROBLEM 10 : Integrate $\int \sqrt{x^2-4}\,dx$.
- PROBLEM 4 : Integrate $\int \frac{\sqrt{x^2+1}}{x}\,dx$.
- PROBLEM 11 : Integrate $\int \frac{x}{\sqrt{x^4-16}}\,dx$.
- PROBLEM 5 : Integrate $\int x^3\sqrt{4-9x^2}\,dx$
- PROBLEM 12 : Integrate $\int \frac{1}{\sqrt{x^2-4x}}\,dx$.
- PROBLEM 6 : Integrate $\int \frac{\sqrt{1-x^2}}{x}\,dx$.
- PROBLEM 13 : Integrate $\int \frac{x}{\sqrt{x^2+4x+5}}\,dx$.
- PROBLEM 7 : Integrate $\int \frac{\sqrt{x^2-9}}{x^2}\,dx$.
- PROBLEM 14 : Integrate $\int x\cdot\sqrt{10x-x^2}\,dx$.
- PROBLEM 8 : Integrate $\int \frac{\sqrt{x^2+1}}{x^2}\,dx$.
- PROBLEM 15 : Integrate $\int \sqrt{\frac{x-1}{x}}\,dx$.
- PROBLEM 9 : Integrate $\int \sqrt{x^2+25}\,dx$.
- PROBLEM 16 : Integrate $\int \sqrt{1-x}\cdot\sqrt{x+3}\,dx$

This is a tricky domain because, unlike simple arithmetic, to solve a calculus problem - and in particular to perform integration - you have to be smart about which integration technique should be used: integration by partial fractions, integration by parts, and so on.

- Marvin Minsky