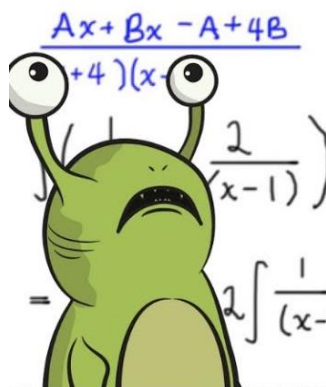


PARTIAL FRACTION DECOMPOSITION

$$\int \frac{3x+7}{x^2+3x-4} dx$$

$$\frac{3x+7}{x^2+3x-4} = \frac{A}{(x+4)}$$



Recommended videos: <https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/unit-4-techniques-of-integration/part-b-partial-fractions-integration-by-parts-arc-length-and-surface-area/session-74-integration-by-partial-fractions/> and <https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/unit-4-techniques-of-integration/part-b-partial-fractions-integration-by-parts-arc-length-and-surface-area/session-75-advanced-partial-fractions/>

**PART I: 1.** Find the *partial fraction decomposition* of each of the following rational functions.  
(To check your answers in Mathematica, use the *Apart* command.)

(a)  $\frac{x+3}{(x+1)(x+2)}$  (b)  $\frac{x^4+1}{x(x-2)}$  (c)  $\frac{1}{(x+5)(x-2)(x-3)}$   
 (d)  $\frac{x^2+1}{(x+4)^2(x-2)}$  (e)  $\frac{x+3}{(x+5)(x-2)(x-3)}$

**2.** Find the *form* of the partial fraction decomposition of each of the following.

You need not solve for the constants.

(a)  $\frac{x+8}{(x+5)^3(x-2)^2(x-3)}$  (b)  $\frac{3x}{(x^2+5)(x^2-7x+10)}$   
 (c)  $\frac{x^5+x+1}{(x^2+x+5)^2(x+2016)}$   
 (d)  $\frac{x^{11}+4x^4+1}{(x^2+1)^3(x-1)(x+2)^4}$

**3.** Compute the *indefinite integral* of each of the following functions.

(Warning: first convert to a rational function!)

(a)  $\frac{1}{e^{2x}-2e^x-35}$  (b)  $\frac{\sin x}{\cos^2 x + \cos x - 20}$  (c)  $\frac{1}{x(\ln x)(1+(\ln x)^2)}$

**Part II:** Fun with University of Michigan test problem.

1. (Math 116 / Exam 1 / October 10, 2007) For this problem,  $\int_1^5 g(x)dx = 12$ , and  $f(x) = 2x - 9$ . Values of  $g(x)$  are given in the table below.

$x$	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

(a) Find  $\int_5^7 g(f(x))dx = 12$ , and  $f(x) = 2x - 9$ .

- (b) Find  $\int_1^5 f(x)g'(x) dx$   
 (c) Find  $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)} dx$ . Hint: begin with the substitution  $w = g(x)$ .

2. (Math 116 / Exam 1 / Fall, 2009) A population of creatures is placed on a small preservation space. Ten creatures are initially placed on the preservation. The time it takes for a population to reach C creatures is given by

$$T(C) = \int_{10}^C \frac{20}{x(400-x)} dx$$

where T is measured in years after the creatures were first placed on the preservation.

- (a) Find a function for T(C) by analytically solving the integral given above. Be sure to show all the appropriate work.  
 (b) How long does it take for the creatures to reach a population of 50? State your answer in a complete sentence and include units in your answer.
3. (Math 116 / Exam 1 / Fall, 2014) For each of the following compare the two given quantities by writing “>”, “<”, “=”, or “NI”(for not enough information).
- (a) Suppose that f(x) is continuous and positive. Compare  $\int_0^1 f(x) dx$  to  $\int_0^1 xf(x^2) dx$ .  
 (b) Suppose that  $\int \frac{1}{(x+2)(x-1)} dx = \int \left( \frac{C}{x+2} + \frac{D}{x-1} \right) dx$ . Compare C to D.  
 (c) Let  $f(x) = x^2$ . Let A be the average value of f(x) over the interval [7, 8], and let  $B = \frac{f(13)}{3}$ . Compare A to B.  
 (d) Let h(x) be a continuous function and let H(x) and G(x) be two anti-derivatives of h(x). Suppose that  $H(0) > G(0)$ . Compare G(1) to H(1).  
 (e) Let  $F(x) = \int_0^x f(t) dt$  where f(t) is increasing and positive. Compare F(1) to F'(1).



*We live but a fraction of our lives.*

- [Henry David Thoreau](#)

*This is a tricky domain because, unlike simple arithmetic, to solve a calculus problem - and in particular to perform integration - you have to be smart about which integration technique should be used: integration by partial fractions, integration by parts, and so on.*

- [Marvin Minsky](#) (creator of artificial neural networks)

