

**Review:**

- 1) Find the *form* of the partial fraction decomposition of each of the following. You need not solve for the constants.

(a) $\frac{x+8}{(x+5)^3(x-2)^2(x-3)}$ (b) $\frac{3x}{(x^2+5)(x^2-7x+10)}$ (c) $\frac{x^5+x+1}{(x^2+x+5)^2(x+2016)}$

(d) $\frac{x^{11}+4x^4+1}{(x^2+1)^3(x-1)(x+2)^4}$

- 2) Compute the *indefinite integral* of each of the following functions.

(Warning: first convert to a rational function!)

(a) $\frac{1}{e^{2x}-2e^x-35}$ (b) $\frac{\sin x}{\cos^2 x + \cos x - 20}$ (c) $\frac{1}{x(\ln x)(1+(\ln x)^2)}$

❖ Fun with University of Michigan test problems.

- 3) (Math 116 / Exam 1 / October 10, 2007) For this problem, $\int_1^5 g(x)dx = 12$, and $f(x) = 2x - 9$. Values of $g(x)$ are given in the table below.

x	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

- (a) Find $\int_5^7 g(f(x))dx = 12$, and $f(x) = 2x - 9$.
- (b) Find $\int_1^5 f(x)g'(x) dx$
- (c) Find $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)} dx$. Hint: begin with the substitution $w = g(x)$.
- 4) (Math 116 / Exam 1 / Fall, 2009) A population of creatures is placed on a small preservation space. Ten creatures are initially placed on the preservation. The time it takes for a population to reach C creatures is given by

$$T(C) = \int_{10}^C \frac{20}{x(400-x)} dx$$

where T is measured in years after the creatures were first placed on the preservation.

- (a) Find a function for $T(C)$ by analytically solving the integral given above. Be sure to show all the appropriate work.
- (b) How long does it take for the creatures to reach a population of 50? State your answer in a complete sentence and include units in your answer.
- 5) (Math 116 / Exam 1 / Fall, 2014) For each of the following compare the two given quantities by writing “>”, “<”, “=” or “NI” (for not enough information).
- (a) Suppose that $f(x)$ is continuous and positive. Compare $\int_0^1 f(x) dx$ to $\int_0^1 xf(x^2) dx$.

- (b) Suppose that $\int \frac{1}{(x+2)(x-1)} dx = \int \left(\frac{C}{x+2} + \frac{D}{x-1} \right) dx$. Compare C to D.
- (c) Let $f(x) = x^2$. Let A be the average value of $f(x)$ over the interval $[7, 8]$, and let $B = \frac{f(13)}{3}$. Compare A to B.
- (d) Let $h(x)$ be a continuous function and let $H(x)$ and $G(x)$ be two anti-derivatives of $h(x)$.
Suppose that $H(0) > G(0)$. Compare $G(1)$ to $H(1)$.
- (e) Let $F(x) = \int_0^x f(t) dt$ where $f(t)$ is increasing and positive. Compare $F(1)$ to $F'(1)$.

Mixed techniques (Stewart, section 7.5). Evaluate each integral.

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|---|--|---|--|
| 1. $\int \frac{\cos x}{1 - \sin x} dx$
Answer | 12. $\int \frac{2x-3}{x^3+3x} dx$ | 25. $\int_0^1 \frac{1+12t}{1+3t} dt$
Answer | 38. $\int_{\pi/6}^{\pi/3} \frac{\sin \theta \cot \theta}{\sec \theta} d\theta$ |
| 2. $\int_0^1 (3x+1)^{\sqrt{3}} dx$ | 13. $\int \sin^5 t \cos^4 t dt$
Answer | 26. $\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1} dx$ | 39. $\int \frac{\sec \theta \tan \theta}{\sec^2 \theta - \sec \theta} d\theta$
Answer |
| 3. $\int_1^4 \sqrt{y} \ln y dy$
Answer | 14. $\int \ln(1+x^2) dx$ | 27. $\int \frac{dx}{1+e^x}$
Answer | 40. $\int_0^\pi \sin 6x \cos 3x dx$ |
| 4. $\int \frac{\sin^3 x}{\cos x} dx$ | 15. $\int x \sec x \tan x dx$
Answer | 28. $\int \sin \sqrt{at} dt$ | 41. $\int \theta \tan^2 \theta d\theta$
Answer |
| 5. $\int \frac{t}{t^4+2} dt$
Answer | 16. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}} dx$ | 29. $\int \ln(x + \sqrt{x^2-1}) dx$
Answer | 42. $\int \frac{\tan^{-1} x}{x^2} dx$ |
| 6. $\int_0^1 \frac{x}{(2x+1)^3} dx$ | 17. $\int_0^\pi t \cos^2 t dt$
Answer | 30. $\int_{-1}^2 e^x - 1 dx$ | 43. $\int \frac{\sqrt{x}}{1+x^3} dx$
Answer |
| 7. $\int_{-1}^1 \frac{e^{\arctan y}}{1+y^2} dy$
Answer | 18. $\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}} dt$ | 31. $\int \sqrt{\frac{1+x}{1-x}} dx$
Answer | 44. $\int \sqrt{1+e^x} dx$ |
| 8. $\int t \sin t \cos t dt$ | 19. $\int e^{x+e^x} dx$
Answer | 32. $\int_1^3 \frac{e^{3/z}}{x^2} dx$ | 45. $\int x^5 e^{-x^3} dx$
Answer |
| 9. $\int_2^4 \frac{x+2}{x^2+3x-4} dx$
Answer | 20. $\int e^2 dx$ | 33. $\int \sqrt{3-2x-x^2} dx$
Answer | 46. $\int \frac{(x-1)e^x}{x^2} dx$ |
| 10. $\int \frac{\cos(1/x)}{x^3} dx$ | 21. $\int \arctan \sqrt{x} dx$
Answer | 34. $\int_{\pi/4}^{\pi/2} \frac{1+4 \cot x}{4-\cot x} dx$ | 47. $\int x^3(x-1)^{-4} dx$
Answer |
| 11. $\int \frac{1}{x^3 \sqrt{x^2-1}} dx$ | 22. $\int \frac{\ln x}{x \sqrt{1+(\ln x)^2}} dx$ | 35. $\int_{-\pi/2}^{\pi/2} \frac{x}{1+\cos^2 x} dx$
Answer | 48. $\int_0^1 x \sqrt{2-\sqrt{1-x^2}} dx$ |
| | 23. $\int_0^1 (1+\sqrt{x})^6 dx$
Answer | 36. $\int \frac{1+\sin x}{1+\cos x} dx$ | 49. $\int \frac{1}{x\sqrt{4x+1}} dx$ |
| | 24. $\int (1+\tan x)^2 \sec x dx$ | 37. $\int_0^{\pi/4} \tan^3 \theta \sec^2 \theta d\theta$ | |