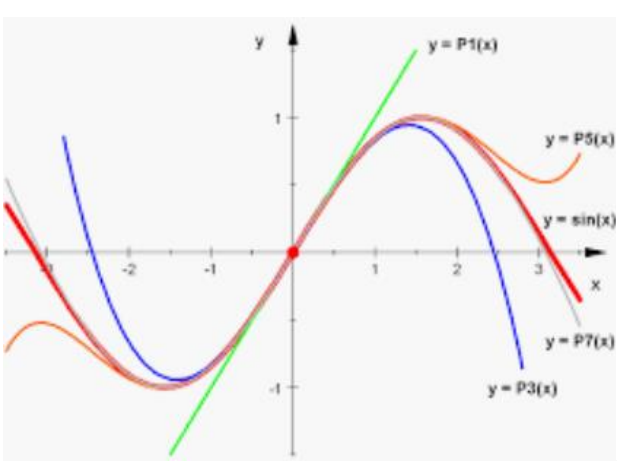
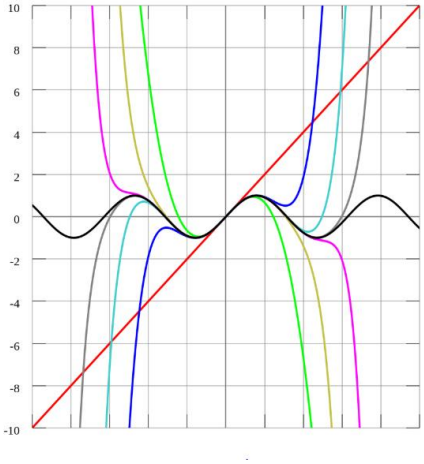
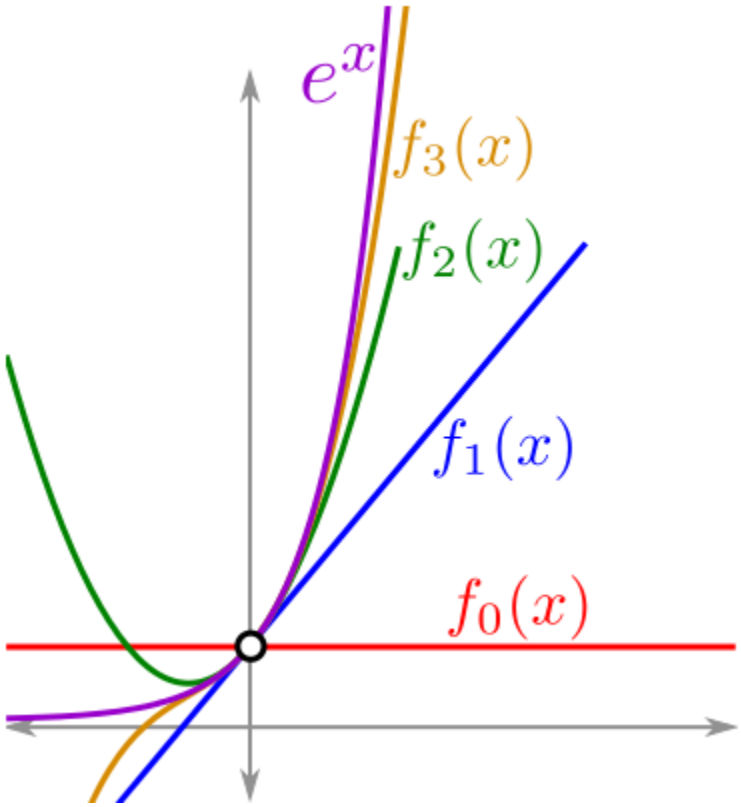
MATH 162 CLASS DISCUSSION

9TH MARCH 2020

TAYLOR POLYNOMIALS, TAYLOR SERIES







1. Find the 5th degree Maclaurin polynomial of e^{3x}.
2. Find the 4th degree Maclaurin polynomial of $(1 - x)\, e^x$.
3. Find the 3rd degree Taylor polynomial of $\frac{1}{1+x^2}$ centered at c = 1.
4. Find the 5th degree Maclaurin polynomial of $\frac{3x-\sin(3x)}{x^3}$.
5. Find the first four *non-zero* terms of the Maclaurin series of $\exp(x^2 + x)$.

6. Write the Maclaurin series expansion for $\frac{x}{1+x^2}$ and for $\ln(1 + x^2)$. Find the interval of convergence for each series. What is the relationship between these two series?

7. Using an appropriate power series expansion, compute $\sum \frac{n}{7^n}$.
(*Hint:* Differentiate an appropriate geometric series.)

8. Find the Maclaurin series of each of the functions:
$2/(3 – x)$, $5/(4 – x)$, and $(23 – 7x)/ [(3 – x)(4 – x)]$.

9. Find the 99^{th} derivative of $1/(a – bx)$ by using an appropriate power series.

10. Find the ***binomial expansion*** of $(1 + x)^{-4}$. What is its radius of convergence?

11. Find the Maclaurin series expansion of $1/(1 + x^2)^{1/2}$.

12. Find the 23^{rd} derivative of $1/(1 + x^2)^{1/2}$.

13. Using an appropriate Maclaurin series, evaluate the limit of $\frac{\sin x - \tan x}{x^3}$ $as\ x \to 0$ without using l'Hôpital's rule.

14. Evaluate the limit of $\frac{\ln x}{x-1}$ $as\ x \to 1$ without using l'Hôpital's rule.

15. Evaluate the limit of $\frac{1}{\sin x} - \frac{1}{x}$ as $x \to 0$ without using l'Hôpital's rule.

16. Evaluate the limit of $\frac{\sin x - x}{\tan x - x}$ as $x \to 0$ without using l'Hôpital's rule.

17. Evaluate the limit of $\frac{\ln x}{e^x - e}$ as $x \to 1$ without using l'Hôpital's rule.
(Hint: Let $t = x – 1$.)

18. Find $\lim_{x \to 0} \frac{e^{x^2} - 1}{\cosh(3x) - 1}$ without using l'Hôpital's rule.

19. State Taylor's inequality. Using this inequality, prove that the Maclaurin series of e^x, sin x, cos x, and cosh x each converge to the given function everywhere.

Colin Maclaurin (1698 – 1746)

Brook Taylor (1685 - 1731)