MATH 162 SOLUTIONS: QUIZ I 24 JANUARY 2020

1. *[10 pts]* Evaluate the following indefinite integral. Show your work!

$$\int sin^3(5x)\cos(5x)\,dx$$

Solution: Since $\frac{d}{dx}\sin(5x) = 5\cos(5x)$, we may set u = sin (5x). Then $\frac{du}{dx} =$

$5\cos(5x)$ *and so* $\cos(5x)\,dx = \frac{du}{5}$.

Consequently,

$$\int sin^3(5x)\cos(5x)\,dx = \int u^3\left(\frac{1}{5}\right)du$$

$$= \frac{1}{5}\int u^3 du = \frac{1}{5}\left(\frac{u^4}{4}\right) + C = \frac{1}{20}\,u^4 + C = \frac{1}{20} sin^4(5x) + C$$

(Alternatively, the method of "judicious guessing" works quite well. Furthermore, splitting up $sin^3(5x)$ into $\left(sin^2(5x)\right)\sin x$ is more time consuming, but will work.)

2. *[10 pts]* Using an appropriate *trigonometric change of variable*, convert the following into a trig integral. No need to evaluate. *Show your work!*

$$\int \frac{x}{(x^2+4)^{\frac{3}{2}}}\,dx$$

Solution: Since the denominator contains an expression of the form $a^2 + x^2$, we choose $x = 2\tan\theta$ to be our substitution. Then $\frac{dx}{d\theta} = 2\sec^2\theta$, we have $dx = 2\sec^2\theta\,d\theta$. Hence

$$\int \frac{x}{(x^2+4)^{\frac{3}{2}}}dx = \int \frac{2\tan\theta}{(4tan^2\theta+4)^{\frac{3}{2}}} 2\sec^2\theta\, d\theta = \int \frac{4\tan\theta\sec^2\theta\, d\theta}{(4(tan^2\theta+1))^{\frac{3}{2}}} =$$

$$\int \frac{4\tan\theta\sec^2\theta\, d\theta}{4^{\frac{3}{2}}(tan^2\theta+1)^{\frac{3}{2}}} = \int \frac{4\tan\theta\sec^2\theta\, d\theta}{8(sec^2\theta)^{\frac{3}{2}}} = \int \frac{4\tan\theta\sec^2\theta}{8(sec^2\theta)^{\frac{3}{2}}} d\theta =$$

$$4\int \frac{\tan\theta\sec^2\theta}{8\sec^3\theta} d\theta = \frac{1}{2}\int \left(\frac{\tan\theta}{\sec\theta}\right) d\theta = \frac{1}{2}\int \tan\theta\cos\theta\, d\theta = \frac{1}{2}\int \sin\theta\, d\theta$$

3. *[10 pts]* Using *integration by parts* compute: $\int_0^1 xe^{3x}\,dx$. *Show your work!*

Solution: Let f(x) = x; consequently, $g'(x)\ must\ equal\ e^{3x}$.

Also, $f'(x) = 1\ and\ g(x) = \frac{1}{3}e^{3x}$.

Using the integration by parts formula, viz.

$$\int_a^b f(x)g'(x)dx = \left(f(x)g(x)\Big|_a^b\right) - \int_a^b f'(x)g(x)dx$$

we have $\int_0^1 xe^{3x}\,dx = \left(x\frac{1}{3}e^{3x}\Big|_0^1\right) - \int_0^1 \frac{1}{3}\ e^{3x}dx = \frac{1}{3}\ (e^3-0) - \frac{1}{9}\ (e^3-1) = \frac{2e^3+1}{9} \approx 2.34$

EXTRA CREDIT (MIT integration bee) *[4 pts each]*

(a) $\int x^{\frac{1}{4}} \ln x \, dx$

Solution: Using *integration by parts*, we let $f(x) = \ln x \; and \; g'(x) = x^{\frac{1}{4}}$

Then $f'(x) = \frac{1}{x} \; and \; g(x) = \frac{4}{5}x^{\frac{5}{4}}$.

Hence $\int x^{\frac{1}{4}} \ln x \, dx = \frac{4}{5}x^{\frac{5}{4}} \ln x - \int \frac{1}{x} \frac{4}{5}x^{\frac{5}{4}} \, dx = \frac{4}{5}x^{\frac{5}{4}} \ln x - \frac{4}{5}\int x^{\frac{1}{4}} dx =$

$$\frac{4}{5}x^{\frac{5}{4}} \ln x - \frac{4}{5}\left(\frac{4}{5}x^{\frac{5}{4}}\right) + C = \frac{4}{5}x^{\frac{5}{4}} \ln x - \frac{16}{25}\, x^{\frac{5}{4}} + C$$

(b) $\int \frac{x}{(2-x)^3} dx$

Solution: This can be achieved with either substitution or partial fraction decomposition. Here is a solution using substitution.

$Let\ z = 2 - x.\ Then\ dz = \ -dx, and\ x = 2 - z.$ Thus

$$\int \frac{x}{(2-x)^3} dx = \int \frac{2-z}{z^3}(-dz) = \int \frac{z-2}{z^3} dz =$$

$$\int (z^{-2} - 2z^{-3}) dz = -\frac{1}{z} + \frac{1}{z^2} = -\frac{1}{2-x} + \frac{1}{(2-x)^2} + C$$


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Or is it whom?