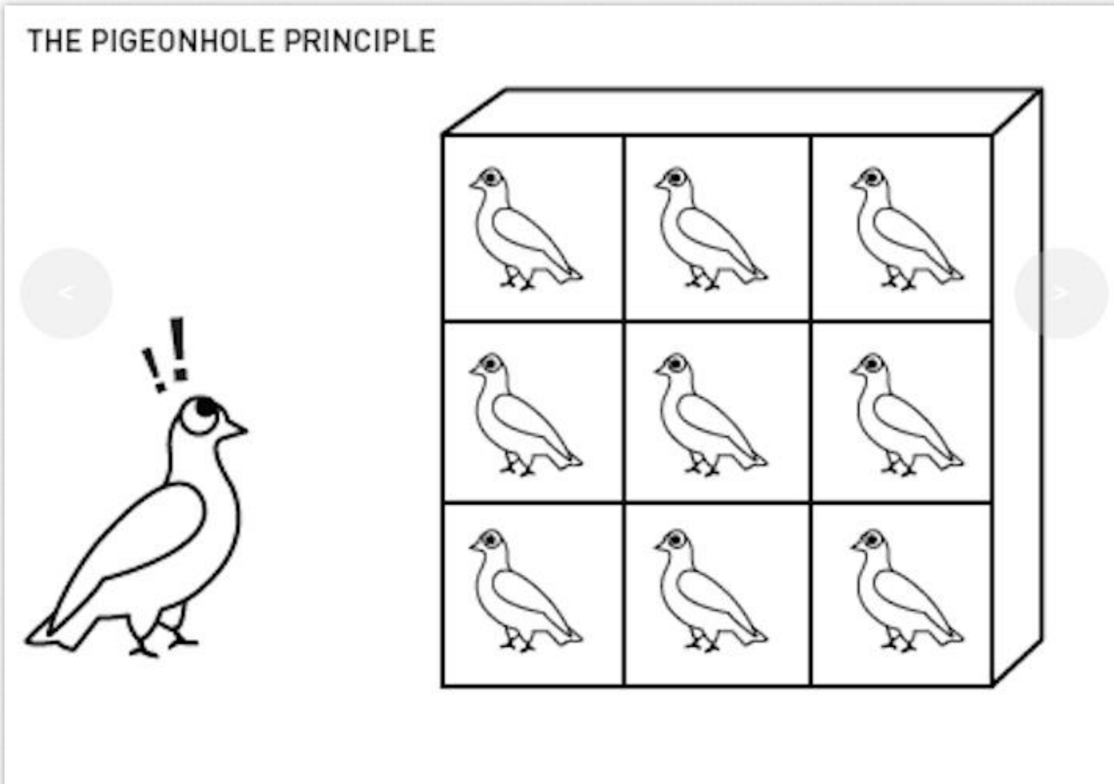
MATH 201: CLASS DISCUSSION (A)   20 FEBRUARY 2020

PIGEON-HOLE PRINCIPLE

INCLUSION-EXCLUSION PRINCIPLE



I State the basic pigeon-hole principle. State the "strong" pigeon-hole principle (n pigeon holes, and at least $kn+1$ pigeons).

II Solve each of the following problems by using the pigeon-hole principle.

A. A bag contains balls of five colors: blue, purple, black, green, and red. What is the *smallest* number of balls that must be drawn from the bag (without looking) so that among the drawn balls, there are at least two of the same color? *(Who are the pigeons, and what are the pigeon-holes?)*

B. Fifteen students in French 103 were given a dictation quiz. Albertine made 13 errors. Each of the other students made fewer errors. Prove that *at least two* students made the same number of errors.

C. There are 30 students in Spanish 103. On a dictation quiz, Carlos made 13 errors, and all the other students made fewer errors. Prove that *at least three* students made the same number of errors.

D. Given 12 integers, show that two of them can be chosen whose difference is divisible by 11. (*Hint:* Let the pigeons represent the twelve integers, and let the numbers mod 11 be the pigeon holes.)

E. Show that among n positive integers, there exist two whose difference is divisible by $n-1$.

F. Prove that for any m positive integers, the sum of some of these integers (possibly only one number) is divisible by m.

G. There are 50 people in a room. Some of them are acquainted with each other, some not. (Assume that "acquainted with" is a symmetric relation.) Prove that two persons in the room have an equal number of acquaintances. (*Hint:* Let the pigeons be the 50 people. Consider two cases: Either everyone is acquainted with at least one other person, or else at least one person has no acquaintances.)

H. 51 points were placed, in an arbitrary way, into a unit square. Prove that one can find 3 of these points that are contained in a circle of radius 1/7. (*Hint:* Cut the square into 25 equal sub-squares.)

Johann Peter Gustav Lejeune Dirichlet first stated the pigeon-hole principle (also known as *Dirichlet's box principle*) in 1834

INCLUSION EXCLUSION PRINCIPLE

1. There are 200 first-year students, 100 of whom are taking Calculus, and 70 of whom are taking Algebra. Fifty first-year students are enrolled in both Calculus and Algebra. How many first-year students are taking neither course?
2. In a survey on the chewing gum preferences of soccer players, it was found that 22 like fruit, 25 like spearmint, 39 like grape, 9 like spearmint and fruit, 17 like fruit and grape, 20 like spearmint and grape, 6 like all flavors, 4 like none. How many players were surveyed?
3. How many integers between 1 and 10000 (inclusive) are divisible by 3 or 5?
   How many are divisible by 3, 5, or 7?
4. Of a hundred volunteers who filled out questionnaires about their viewing habits over the past year: (i) 28 watched gymnastics (ii) 29 watched baseball (iii) 19 watched soccer (iv) 14 watched gymnastics and baseball (v) 12 watched baseball and soccer (vi) 10 watched gymnastics and soccer (vii) 8 watched all three sports. How many watched none of the three sports?
5. In a mathematics department of size 40, faculty are members of four different professional organizations: NCTM (N), MAA (M), AMS (A), and SSMA (S). We know that 21 are members of NCTM, 26 are MAA members, 19 belong to AMS, and 17 are in SSMA. Also, we know that 15 are in both NCTM and MAA, 6 are in NCTM and AMA, 9 are in NCTM and SSMA, 14 are in MAA and AMS, 10 are in MAA and SSMA, and 11 are in AMS and SSMA. We also know that 6 are in NCTM, MAA, and AMS, 5 are in NCTM, MAA, and SSMA, 4 are in NCTM, AMS, and SSMA, and 9 are in MAA, AMS, and SSMA. Finally, 4 people belong to all four organizations. How many faculty members belong to none of these organizations?
6. A 'derangement' of the sequence 1, 2, 3, 4 is a rearrangement of the sequence such that integer j is not placed in the j^{th} position (where j = 1, 2, 3, 4). How many *derangements* of 1, 2, 3, 4 exist?
   Can you generalize this to a sequence of length 5 or 6? What about *n*?

A lot of widows feel that they have betrayed their spouse by continuing to live. It's deranged thinking. I know that, but that doesn't stop you feeling it.

- Joyce Carol Oates