INTRODUCTION TO SETS

STUDY CAREFULLY SECTIONS 1.1 – 1.4 OF HAMMACK

The following notation is commonly used: **Z** for the set of integers; **N** for the set of positive integers (the "natural numbers"), **Q** for the set of all rational numbers; **R** for the set of all real numbers, and Ø for the empty set.

A. List the elements of each of the following sets: (a) $\{x \in R : x^4 - 1 = 0\}$ (b) $\{x \in Z : -1/3 < x < 5.99\}$

   (c) $\{x \in N \mid x \le 4\}$ (d) $\{$unicorns$\mid$ unicorn lives in Illinois$\}$ (e) $\{\}$ (f) $\{\varphi\}$

   (g) $\{1, \{2\}\}$

B. Write in set notation: (a) $\{4, 9, 16, 25, \ldots\}$ (b) $\{1/1, 1/3, 1/5, 1/7, \ldots\}$ (c) $\{\ldots 1/8, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

C. Determine the cardinality of each set in (1).

D. Find cardinality of each of the following sets:

   $\{Kansas\ City, Phoenix, Sacramento, Denver\}$

   $\{\{$Friday$\}, \{$blue moon$\}, \{$dragonfly$\}\}$

   $\{\{1, 2\}, \{\{7, 0, 3\}\}\}$

   $\{p \in N \mid p\ is\ prime\ and\ p \le 25\}$
   $\{\{1\}, \{2, \{3, 4\}\}, \varphi\}$
   $\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\varphi\}\}$
   $\{x \in Z \mid |x| < 10\}$

E. Sketch the following sets of points in the xy-plane.
   $\{(x, y) : x, y \in R, x^2 + y^2 \le 1\}$
   $\{(x, y) : x, y \in R, y \ge x^2 - 1\}$
   $\{(x, y) : x, y \in R, x > 1\}$
   $\{(x, x+y) : x \in R, y \in Z\}$
   $\{(x, \frac{x^2}{y}) : x \in R, y \in N\}$
   $\{(x, y) \in R^2 : (y-x)(y+x) = 0\}$
   $\{(x, y) \in R^2 : (y-x^2)(y+x^2) = 0\}$

1. Suppose $A = \{1,2,3,4\}$ and $B = \{a,c\}$.

(a) $A \times B$	(c) $A \times A$	(e) $\emptyset \times B$	(g) $A \times (B \times B)$
(b) $B \times A$	(d) $B \times B$	(f) $(A \times B) \times B$	(h) B^3

2. Suppose $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$.

(a) $A \times B$	(c) $A \times A$	(e) $A \times \emptyset$	(g) $A \times (B \times B)$
(b) $B \times A$	(d) $B \times B$	(f) $(A \times B) \times B$	(h) $A \times B \times B$

3. $\{x \in R : x^2 = 2\} \times \{a, c, e\}$
4. $\{n \in Z : 2 < n < 5\} \times \{n \in Z : |n| = 5\}$
5. $\{x \in R : x^2 = 2\} \times \{x \in R : |x| = 2\}$
6. $\{x \in R : x^2 = x\} \times \{x \in N : x^2 = x\}$
7. $\{\emptyset\} \times \{0, \emptyset\} \times \{0, 1\}$
8. $\{0, 1\}^4$

F.

Sketch these Cartesian products on the *x-y* plane $\mathbb{R}^2$ (or $\mathbb{R}^3$ for the last two).

9. $\{1,2,3\} \times \{-1,0,1\}$

10. $\{-1,0,1\} \times \{1,2,3\}$

11. $[0,1] \times [0,1]$

12. $[-1,1] \times [1,2]$

13. $\{1,1.5,2\} \times [1,2]$

14. $[1,2] \times \{1,1.5,2\}$

15. $\{1\} \times [0,1]$

16. $[0,1] \times \{1\}$

17. $\mathbb{N} \times \mathbb{Z}$

18. $\mathbb{Z} \times \mathbb{Z}$

19. $[0,1] \times [0,1] \times [0,1]$

20. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\} \times [0,1]$

G. Find the power set of each of the following sets:

1. $\{1,2,3,4\}$

2. $\{1,2,\emptyset\}$

3. $\{\{\mathbb{R}\}\}$

4. $\emptyset$

5. $\{\emptyset\}$

6. $\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$

7. $\{\mathbb{R},\{\mathbb{Q},\mathbb{N}\}\}$

8. $\{\{0,1\},\{0,1,\{2\}\},\{0\}\}$

9. in general if $|S| = n$, what is $|P(S)|$?

H.

Write out the following sets by listing their elements between braces.

9. $\{X : X \subseteq \{3,2,a\}$ and $|X| = 2\}$

10. $\{X \subseteq \mathbb{N} : |X| \le 1\}$

11. $\{X : X \subseteq \{3,2,a\}$ and $|X| = 4\}$

12. $\{X : X \subseteq \{3,2,a\}$ and $|X| = 1\}$

Decide if the following statements are true or false. Explain.

13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$

14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$

15. $\{(x,y) : x - 1 = 0\} \subseteq \{(x,y) : x^2 - x = 0\}$

16. $\{(x,y) : x^2 - x = 0\} \subseteq \{(x,y) : x - 1 = 0\}$

I.

Find the indicated sets.

1. $\mathscr{P}(\{\{a,b\},\{c\}\})$

2. $\mathscr{P}(\{1,2,3,4\})$

3. $\mathscr{P}(\{\{\emptyset\},5\})$

4. $\mathscr{P}(\{\mathbb{R},\mathbb{Q}\})$

5. $\mathscr{P}(\mathscr{P}(\{2\}))$

6. $\mathscr{P}(\{1,2\}) \times \mathscr{P}(\{3\})$

7. $\mathscr{P}(\{a,b\}) \times \mathscr{P}(\{0,1\})$

8. $\mathscr{P}(\{1,2\} \times \{3\})$

9. $\mathscr{P}(\{a,b\} \times \{0\})$

10. $\{X \in \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$

11. $\{X \subseteq \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$

12. $\{X \in \mathscr{P}(\{1,2,3\}) : 2 \in X\}$

Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

13. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))|$

14. $|\mathscr{P}(\mathscr{P}(A))|$

15. $|\mathscr{P}(A \times B)|$

16. $|\mathscr{P}(A) \times \mathscr{P}(B)|$

17. $|\{X \in \mathscr{P}(A) : |X| \le 1\}|$

18. $|\mathscr{P}(A \times \mathscr{P}(B))|$

19. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A \times \emptyset)))|$

20. $|\{X \subseteq \mathscr{P}(A) : |X| \le 1\}|$