MATH 201:   CLASS DISCUSSION, 16 JANUARY 2020

NAÏVE SET THEORY CONTINUED

0. **Review**:

(a) Which of the following statements are ambiguous? Why?

(i) Give me a cake or a pretzel.

(ii) Albertine gave a bath to her dog wearing an orange hat.

(iii) Visiting relatives can be boring.

(iv) I promise that I will give you a ring tomorrow.

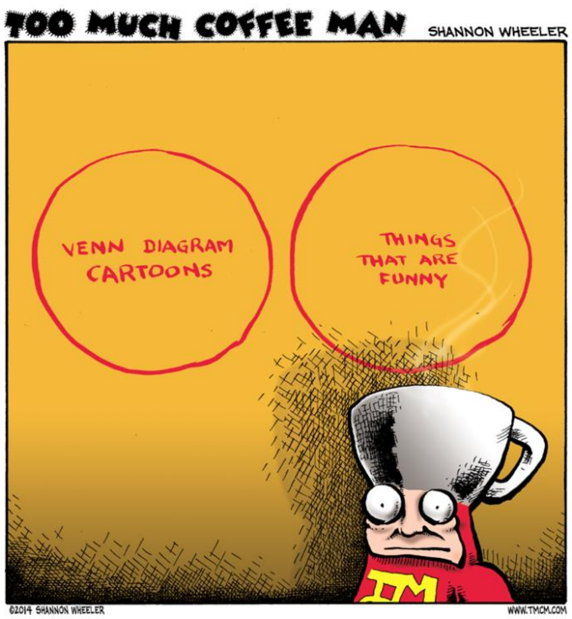
(b) Find the *cardinality* of each of the following sets.

(i) $A = \{x: x \text{ is a prime number and } x < 19\}$.

(ii) $B = \left\{1, 3, \{4, 5, \{2020\}\}\right\}$

(iii) $C = \emptyset$

(c) What is meant by the term *Cartesian product*? Give examples (cf, cd 14 Jan, part B)



1. What is meant by the term *subset*?

2. What is meant by the term *power set* of a set? Consider examples (cf, cd 14 Jan, part B)

3. Let L be a finite set. Find the cardinality of $\mathscr{P}$(L) if the cardinality of L is

(a) 0 (b) 1 (c) 2 (d) 3 (e) n, where n is a non-negative integer.

4. Let A, B and C be three sets such that:

A = {2, 4, 6, 8, 10, 12}, B = {3, 6, 9, 12, 15} and C = {1, 4, 7, 10, 13, 16}. Find:

(i) $A \cup B$ (ii) $A \cap B$ (iii) $B \cap A$ (iv) $B \cup A$ (v) $B \cup C$ (vi) $A - B$

(vii) $A - (B \cup C)$ (viii) $A - (B \cap C)$

(ix) Is $A \cup B = B \cup A$? (x) Is $B \cap C = B \cup C$?

5. Let A be a subset of a universe X. What is meant by the term *complement of A*? This is denoted by $\bar{A}$. (Note that is many other textbooks and websites, the alternative notation, A^c, may be used.)

6. Complete each of the following. Use a Venn diagram to justify each answer.

(a) Associativity of union

$A \cup (B \cup C) =$

(b) **Associativity of intersection**

$A \cap (B \cap C) =$

(c) Commutativity

(i) $A \cup B =$

(ii) $A \cap B =$

(d) Double complement

$\overline{(\bar{A})}$

(e) Complementation

(i) $A \cup \bar{A} =$

(ii) $A \cap \bar{A} =$

7. *True or False?* Give a general argument or a *counterexample*.

(a) $A \cup B \subseteq A \cap B$

(b) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(c) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

(d) $A - (B \cap C) = (A - B) \cup (A - C)$

(e) $A - B = B^c - A^c$

(f) $(A \cup B) \cap C \supseteq (A \cup B) \cap (A \cup C)$

(g) $\mathcal{P}(E) \cap \mathcal{P}(F) = \mathcal{P}(E \cap F)$

(h) $\mathcal{P}(E) \cup P(F) \subseteq \mathcal{P}(E \cup F)$