MATH 201: CLASS DISCUSSION, 21 JANUARY 2020

NAÏVE SET THEORY CONTINUED

Venn diagrams were introduced in 1880 by **John Venn**.
They are used to teach elementary set theory.
Venn diagrams are also used to illustrate simple set relationships in
probability, logic, statistics, linguistics, and computer science.

(1) What is meant by the term *subset*?

(2) What is meant by the term *power set* of a set? Consider examples (cf, cd 14 Jan, part B)

(3) Let L be a finite set. Find the cardinality of $\wp(L)$ if the cardinality of L is

(a) 0 (b) 1 (c) 2 (d) 3 (e) n, where n is a non-negative integer.

(4) Let A, B and C be three sets such that:

A = {2, 4, 6, 8, 10, 12}, B = {3, 6, 9, 12, 15} and C = {1, 4, 7, 10, 13, 16}. Find:

(i) A ∪ B (ii) A ∩ B (iii) B ∩ A (iv) B ∪ A (v) B ∪ C (vi) A – B

(vii) A – (B ∪ C) (viii) A – (B ∩ C)

(ix) Is A ∪ B = B ∪ A? (x) Is B ∩ C = B ∪ C?

(5) Let A be a subset of a universe X. What is meant by the term *complement of A*? This is denoted by $\bar{A}$. (Note that is many other textbooks and websites, the alternative notation, A^c, may be used.)

(6) Complete each of the following. Use a Venn diagram to justify each answer.

(a) Associativity of union
$A \cup (B \cup C) =$
(b) **Associativity of intersection**
$A \cap (B \cap C) =$
(c) Commutativity
(i) $A \cup B =$
(ii) $A \cap B =$
(d) Double complement
$\overline{(\bar{A})} =$
(e) Complementation
(i) $A \cup \bar{A} =$
(ii) $A \cap \bar{A} =$

(7) *True or False?* If true, give a Venn diagram argument. If false, provide a counterexample.

(a) $A \cup B \subseteq A \cap B$ (b) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(c) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

(d) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

(e) $A - B = \overline{B} - \overline{A}$

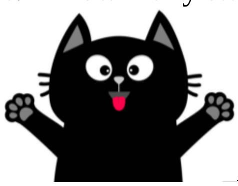
(f) $A - (B \cap C) = (A - B) \cup (A - C)$

(g) $(A \cup B) \cap C \supseteq (A \cup B) \cap (A \cup C)$

(8) *True or False:* (a) $\mathcal{P}(E) \cap \mathcal{P}(F) = \mathcal{P}(E \cap F)$ (b) $\mathcal{P}(E) \cup P(F) \subseteq \mathcal{P}(E) \cup P(F)$

(9) Out of forty students, 14 are taking English Composition, and 29 are taking Chemistry.

a. If five students are in both classes, how many students are in neither class?
b. How many are in each class?
c. How many students are taking only Chemistry?



(10) A vet is examining cats to figure out what is causing an outbreak of a mysterious feline illness. The technician surveyed 87 customers that week. They were asked (1) if the cats were permitted outside, and (2) if the household also has a dog. There were 30 people who said "no" to both questions, and 20 answered "yes" to the outdoors question. While 47 answered "yes" to the dog question, the technician forgot to record how many answered "yes" to both questions. How many was that?

(11) In a group of 39 students, there are 23 who like the Impossible Burger, and the others do not. All the students were asked if they are taking Math or Physics. The responses were:

- 19 are taking Math
- 16 are taking Physics
- 5 who dislike the Impossible Burger are taking Math and Physics
- 9 who like the Impossible Burger are taking only Math
- 1 who dislikes the Impossible Burger takes Physics only
- 3 who like the Impossible Burger are taking both Math and Physics

a) How many Impossible Burger lovers take Physics?

b) How many students are taking only Math or only Physics?

(12) All students have to take at least one math class and one language class. Twenty students take calculus, and thirty students take statistics. Fifteen students take Spanish, and twenty-five students take French. If there are thirty-five students total, what is the maximum number of students taking both two math classes and two language classes?

(13) State and explain using Venn diagrams **de Morgan's laws,** viz.

$\overline{(\boldsymbol{A} \cup \boldsymbol{B})} =$ $\overline{(\boldsymbol{A} \cap \boldsymbol{B})} =$

(14) *True or False?* Here $\mathcal{P}(X)$ denotes the power set of X.

(a) $\mathcal{P}(E) \cap \mathcal{P}(F) = \mathcal{P}(E \cap F)$

(b) $\mathcal{P}(E) \cup \mathcal{P}(F) \subseteq \mathcal{P}(E \cup F)$

(15) Define the **Cartesian product** of two (or more) sets. Let A = $\{0, \beta\}$ and B = $\{9, c\}$. List the elements of each of the following sets, using braces for the collection of elements.

(a) $A \times B$ (b) B×C (c) $A \times A$ (d) $B \times B$ (e) $\emptyset \times B$

(f) $(A \times B) \times B$ (g) $A \times (B \times B)$

(b) Repeat exercise (14), using A = {c, d, e}, B = $\{\alpha, \beta\}$, and C ={7, 11}.

(c) Sketch the following subsets of $\mathbf{R}^2$. (i) $\{1, 2\} \times \{3, 5\}$ (ii) $[0, 2] \times [0, 2]$

(iii) $[0, 1] \times \{5\}$ (iv) $Z \times Z$ (v) $N \times Z$

(vi) $R \times [-1, 1]$ (vii) {(x, y): x > y + 1} (viii) $\{(x, y):\ 1 \leq x^2 + y^2 \leq 4\}$

(16) What is meant by a collection of **indexed sets**? Perform each of the following operations upon the given indexed set.

Operations: $\bigcup_{i \in I} A_i$ $\bigcap_{i \in I} A_i$

(i) Let I = {1, 2, 3}, A_1 = {1, 3}, A_2 = {2, 3}, A_3 = {2, 3, 4}

(ii) Let I = N, A_n = [n, ∞)

(iii) Let I = Z, A_n = [-n, n]

(iv) Let I = N, $A_n = \left[-\frac{1}{n}, \frac{1}{n}\right]$

(v) Let I = N, $A_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$