CLASS DISCUSSION: 30 JANUARY 2020

FIRST-ORDER PREDICATE LOGIC, CONTINUED

1. For each of the following strings, answer YES or NO, depending upon whether the string of symbols is a wff (well-formed formula) or not. You need not justify your answers.

(a) $(p \to \sim q) \to (r \to (p \wedge q))$
(b) ~p
(c) $pq \to \sim r$
(d) $p\sim$
(e) $(\sim p \wedge q) \to (r \vee (s \to t))$
(f) $p \wedge \to q$

2. *Using truth tables*, determine if the following statements are logically equivalent or not. $p \to (q \wedge \sim q)$ *and* $\sim p$.

3. What is meant by the *converse* of an implication? State the converse of each of the following statements:

(a) If Albertine is happy, then Swann is sad.
(b) If you get a 100 on your final exam, then your professor will give you an "A."
(c) If you live in Austin, then you live in Texas.
(d) If a number is divisible by 10, then the number ends in zero.

4. *Using a biconditional, rewrite the following sentences symbolically*.

(a) A square matrix is invertible if and only if its determinant is not 0.

(b) An integer is divisible by 9 if and only if the sum of its digits is divisible by 9.

(c) A triangle has 3 equal sides if and only if it has 3 equal angles.

(d) Let *m* and *n* be integers. Then $m^3 – n^3$ is even if and only if m – n is even.

5. Let U be the universe of all positive integers greater than 2.
Consider the following predicates:
P(x) = "x is a prime number"   Q(x) = "x is odd"

Express each of the following statements in symbolic form.

(a) "x being prime is a *sufficient* condition for x being odd."
(b) "x being odd is a *necessary* condition for x being prime."
(c) "if x is odd and y is prime, then x + y is not odd."

6. Distribute the negation through each of the following statements.

(a) $\sim \exists x \in A,\ x > 0$
(b) $\sim \forall x \in A,\ x > 0$
(c) $\sim \forall x \in A\ \forall y \in B\ \exists z \in X,\ z > xy$
(d) $\sim \exists p \in X\ \exists q \in B\ \forall r \in C,\ r + p < q$
(e) $\sim \exists c \in C\ \ \forall x \in X\ \ \forall y \in B,\ \ln(1 + c^2) > x^2 + y^4 - 5$
(f) $\sim \exists a, b, c \in Z^+\ \ a^4 + b^4 = c^4$

7. Explain why the following are equivalent:

$\sim \forall x\ P(x)$ and $\sim \exists x \sim P(x)$

Analogously, $\sim \exists x\ P(x)$ and $\forall x \sim P(x)$ are equivalent. Give examples from natural language.

8. Consider the universe of all people in the United States. Define the following predicates:

P(x) = “x is a student”   Q(x) = “x is smart”   L(x, y) = “x loves y”

Express each of the following in symbolic form.

(a) All the students are smart.
(b) There exists a student.
(c) There exists a smart student.
(d) Every student loves some other student
(e) No one loves anyone.
(f) If Albertine loves anyone, then she loves Swann.
(g) There is a student who loves himself but who is loved by *no one else*.

9. Negate each of the following sentences. Your answer should be a sentence in English – not a symbolic sentence. (Nor should you simply write: “It is not true that ….”).

(a) You can fool all of the people all of the time.
(b) If cos x = 7, then it is not true that x is an integer.
(c) If the cat ate the mouse or the mouse ate the flea, then Albertine will either watch Dark on Netflix or go swimming in the lake.
(d) You cannot teach a cat to fetch, but you can teach a dog to swim.

10. State the negation of each of the following quantified propositions.

a) $(\forall x \in S)(x \geq 0)$

b) $(\exists n \in S)(n \text{ is a prime number})$

c) $(\forall x)(\exists y)(xy = 10)$ (which we read “for all x, there exists a y such that $xy = 10$”)

d) $(\exists x)(\forall y)(xy \neq 10)$ (which we read “there exists an x such that for all y we have $xy \neq 10$)

11. Is the statement (p∨~q)∨(~p∧q) a tautology?

➢ *The following are Florida State University questions.*

12. Select the statement that is the negation of "Today is Monday, and it isn't raining."

A. Today isn't Monday, and it isn't raining.
B. Today isn't Monday, or it isn't raining.
C. Today isn't Monday, or it is raining.
D. Today isn't Monday, and it is raining.
E. Today is Friday, and it is snowing.

13. Select the statement that is the negation of "I'm careful, or I make mistakes."

A. I'm not careful, and I don't make mistakes.
B. I'm not careful, or I don't make mistakes.
C. I'm not careful, and I make mistakes.
D. I'm not careful, or I make mistakes.
E. I never make misteaks. *[sic]*

14. Select the statement that is the negation of "I walk, or I chew gum."

A. I don't walk, and I chew gum.
B. I don't walk, or I chew gum.
C. I don't walk, and I don't chew gum.
D. I don't walk, or I don't chew gum.
E. I walk until I step on chewed gum.

15. Select the statement that is the negation of "I'm mad as heck, and I'm not going to take it anymore."

A. I'm not mad as heck, and I'm not going to take it anymore.
B. I'm not mad as heck, or I'm not going to take it anymore.

C. I'm not mad as heck, and I am going to take it anymore.
D. I'm not mad as heck, or I am going to take it anymore.

16. Select the statement that is the negation of "All of the businesses are closed."
A. Some of the businesses are closed.
B. Some of the businesses are not closed.
C. None of the businesses are closed.
D. All of the businesses are open.
E. All of my clothes are businesslike.

17. Select the statement that is the negation of the statement "Some pilots are pirates."
A. All pilots are pirates.
B. No pilots are pirates.
C. Some pilots are not pirates.
D. All pirates are pilots.

18. *'But wait a bit,' the Oysters cried, 'Before we have our chat;*
For some of us are out of breath, And all of us are fat!'
"No hurry!' said the Carpenter. They thanked him much for that.
Select the statement that is the negation of "Some of us are out of breath, and all of us are fat."
A. Some of us aren't out of breath, or none of us is fat.
B. Some of us aren't out of breath, and none of us is fat.
C. None of us is out of breath, and some of us aren't fat.
D. None of us is out of breath, or some of us aren't fat.

19. Select the statement that is the negation of “All summer days are muggy.”
A. All muggy days are summer.
B. Some summer days are muggy.
C. Some summer days are not muggy.
D. No summer days are muggy.

20. Select the statement that is the negation of “Some weasels are cuddly.”
A. No, weasels are cuddly.
B. All weasels are cuddly.
C. Some weasels are not cuddly.
D. All cuttlefish are weasely.

21. Select the statement that is the negation of “Coach Spurrier is charming, and Coach Spurrier is modest.”
A. Coach Spurrier is not charming, and Coach Spurrier is not modest.
B. Coach Spurrier is not charming, or Coach Spurrier is not modest.
C. Coach Spurrier is not charming, and Coach Spurrier is modest.
D. Let’s get serious for a minute.

22. Select the statement that is the negation of “The speed limit is 55 mph, and granny is driving 35 mph.”
A. The speed limit is not 55, or granny is not driving 35.
B. The speed limit is not 55 and granny is not driving 35
C. The speed limit is not 55, or granny is driving 35.
D. The speed limit is not 55, and granny is driving 35.
E. Counseling for road rage is available at 1 - 900 – calm down

REVIEW problems from Hammack:

Exercises for Section 2.3

Without changing their meanings, convert each of the following sentences into a sentence having the form "*If P, then Q*."

1. A matrix is invertible provided that its determinant is not zero.
2. For a function to be continuous, it is sufficient that it is differentiable.
3. For a function to be continuous, it is necessary that it is integrable.
4. A function is rational if it is a polynomial.
5. An integer is divisible by 8 only if it is divisible by 4.
6. Whenever a surface has only one side, it is non-orientable.
7. A series converges whenever it converges absolutely.
8. A geometric series with ratio r converges if $|r| < 1$.
9. A function is integrable provided the function is continuous.
10. The discriminant is negative only if the quadratic equation has no real solutions.
11. You fail only if you stop writing. (Ray Bradbury)

Exercises for Section 2.4

Without changing their meanings, convert each of the following sentences into a sentence having the form "*P if and only if Q*."

1. For matrix A to be invertible, it is necessary and sufficient that $\det(A) \neq 0$.
2. If a function has a constant derivative then it is linear, and conversely.
3. If $xy = 0$ then $x = 0$ or $y = 0$, and conversely.
4. If $a \in \mathbb{Q}$ then $5a \in \mathbb{Q}$, and if $5a \in \mathbb{Q}$ then $a \in \mathbb{Q}$.
5. For an occurrence to become an adventure, it is necessary and sufficient for one to recount it. (Jean-Paul Sartre)

Exercises for Section 2.5

Write a truth table for the logical statements in problems 1–9:

1. $P \vee (Q \Rightarrow R)$
2. $(Q \vee R) \Leftrightarrow (R \wedge Q)$
3. $\sim (P \Rightarrow Q)$
4. $\sim (P \vee Q) \vee (\sim P)$
5. $(P \wedge \sim P) \vee Q$
6. $(P \wedge \sim P) \wedge Q$
7. $(P \wedge \sim P) \Rightarrow Q$
8. $P \vee (Q \wedge \sim R)$
9. $\sim (\sim P \vee \sim Q)$
10. Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (This can be done without a truth table.)
11. Suppose P is false and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S. (This can be done without a truth table.)

Exercises for Section 2.6

A. Use truth tables to show that the following statements are logically equivalent.

1. $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
2. $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
3. $P \Rightarrow Q = (\sim P) \vee Q$
4. $\sim (P \vee Q) = (\sim P) \wedge (\sim Q)$
5. $\sim (P \vee Q \vee R) = (\sim P) \wedge (\sim Q) \wedge (\sim R)$
6. $\sim (P \wedge Q \wedge R) = (\sim P) \vee (\sim Q) \vee (\sim R)$
7. $P \Rightarrow Q = (P \wedge \sim Q) \Rightarrow (Q \wedge \sim Q)$
8. $\sim P \Leftrightarrow Q = (P \Rightarrow \sim Q) \wedge (\sim Q \Rightarrow P)$

B. Decide whether or not the following pairs of statements are logically equivalent.

9. $P \wedge Q$ and $\sim (\sim P \vee \sim Q)$
10. $(P \Rightarrow Q) \vee R$ and $\sim ((P \wedge \sim Q) \wedge \sim R)$
11. $(\sim P) \wedge (P \Rightarrow Q)$ and $\sim (Q \Rightarrow P)$
12. $\sim (P \Rightarrow Q)$ and $P \wedge \sim Q$
13. $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$
14. $P \wedge (Q \vee \sim Q)$ and $(\sim P) \Rightarrow (Q \wedge \sim Q)$

Exercises for Section 2.7

Write the following as English sentences. Say whether they are true or false.

1. $\forall x \in \mathbb{R}, x^2 > 0$
2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$
3. $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$
4. $\forall X \in \mathscr{P}(\mathbb{N}), X \subseteq \mathbb{R}$
5. $\forall n \in \mathbb{N}, \exists X \in \mathscr{P}(\mathbb{N}), |X| < n$
6. $\exists n \in \mathbb{N}, \forall X \in \mathscr{P}(\mathbb{N}), |X| < n$
7. $\forall X \subseteq \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$
8. $\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n$
9. $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 5$
10. $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$