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1 SUMMARY OF PREVIOUS AND CURRENT WORK

My area of research is algebraic topology, in particular, surgery theory of manifolds and quadratic forms. Topology studies the geometric properties of spaces that are preserved by continuous deformations. Manifolds are the main examples of topological spaces, with the local properties of Euclidean space in an arbitrary dimension n. They are the higher dimensional analogs of curves and surfaces. For example, a circle is a one-dimensional manifold. Balloons and doughnuts are examples of two-dimensional manifolds. A balloon cannot be deformed continuously into a doughnut, so we see that there are essential topological differences between them.

An "invariant" of a topological space is a number or an algebraic structure such that topologically equivalent spaces have the same invariant. For example the essential topological difference between the balloon and the doughnut is calculated by the "Euler characteristic", which is 2 for a balloon and 0 for a doughnut.

In my thesis [Rov15], which I recently completed at the University of Edinburgh under the supervision of Prof. Andrew Ranicki, I investigate the relation between three different but related invariants of manifolds with dimension divisible by 4: the signature, the Brown-Kervaire invariant and the Arf invariant.

The *signature invariant* takes values in the set $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ of integers. In my thesis I focus on the signature invariant modulo 8, that is, its remainder after division by 8.

The *Brown-Kervaire invariant* takes values in the set $\{0,1,2,3,4,5,6,7\}$.

The *Arf invariant* takes values in the set $\{0,1\}$.

One of the main results in my thesis uses the Brown-Kervaire invariant to prove that for a manifold with signature divisible by 4, the divisibility by 8 is decided by the Arf invariant. This result is then applied to understand the signatures of fibre bundles. A *fibre bundle* is a way to take "twisted products" of topological spaces and thus to construct a more complicated space from two simpler spaces.

The problem of understanding the signatures of fibre bundles has a distinguished lineage: Chern, Hirzebruch and Serre [CHS57] were the first to consider the problem of non-multiplicativity of the signature of a fibre bundle. Kodaira [Kod67], Hirzebruch [Hir69] and Atiyah [Ati69] independently constructed examples of fiber bundles with non-trivial signature. Meyer [Mey73] proved that the signature of a surface bundle is in general divisible by 4. Later on, Hambleton, Korzeniewski and Ranicki [HKR07] provided a high-dimensional version of this result. Currently in Cambridge both Dr Oscar Randal-Williams and Prof Ivan Smith have worked on different aspects of the problem of signatures of fibre bundles. In particular Dr Randal-Williams has expressed his interest in my work.

My thesis is entirely concerned with pure mathematics. However it is possible that it has applications in mathematical physics, where the signature modulo 8 plays a significant role. Atiyah [Ati90] pointed out that finding non-trivial signatures in the context of fibre bundles is closely related to the problem of the construction of Topological Quantum Field Theories (TQFTs).

2 Research Objectives and Methods

The main goal of my proposed research is to study the obstructions to divisibility of the signature by higher powers of 2 using certain mathematical theories, namely L-theory and K-theory. This main idea will be complemented by the construction of examples illustrating the theoretical results, and will then be applied to the construction of Topological Quantum Field Theories (TQFTs).

The project will have the following three **main objectives (O.1-3)**:

O1. Study the non-multiplicativity properties of the signature of fibre bundles modulo higher powers of 2.

The signature of fibre bundles has now been studied modulo 2, 4 and 8. The study of the signature modulo higher powers of 2 will produce relevant results in the area, as well as give answers to deep questions posed by Rognes, Weiss and Levikov on the relation between the signature modulo higher powers of 2 and the higher K-theory groups of $\mathbb{Z}$.

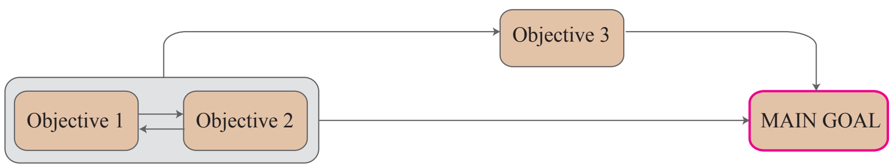
O2. Construct high-dimensional examples of fibre bundles with non-multiplicative signature.

One major problem in the context of the non-multiplicativity of the signature is to find non-trivial examples of bundles with non-zero signatures. The idea leading to this objective was suggested by Oscar Randal-Williams, concerning fibre bundles with high dimensional fibres.

O3. Apply the results about non-multiplicativity of the signature of fibre bundles to the construction of TQFTs.

As explained by Banagl [Ban13], Novikov and Atiyah pointed out that the additivity property of the signature is equivalent to building a non-trivial topological quantum field theory. A greater understanding of the properties of signatures of fibre bundles would allow for advances in the question of the construction of TQFTs.

The following diagram highlights the relationships between the objectives:



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