MATH FOR A CHANGE

Revised 2012

Mathematics Teachers' Association

Math for a Change

2012 Revised Edition

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Introduction

Mathematics. Social justice. To many people, these sound like quite different domains – like a Venn Diagram with no intersection. After all, what do crime and the justice system have to do with arithmetic or statistics? Why would we talk about poverty in a Geometry class? Or pay day loan companies in a Pre-calculus class? Don't those topics belong in social studies?

Fortunately this schism between social justice issues and high school mathematics was addressed by Fr. Robert Thul (St. Ignatius College Prep, Chicago) and Kevin Mistrik (Loyola Academy) in the early 1990's. Published by MTA as *Math For a* Change (1993) their lessons on social justice were used for many years in mathematics classes at their schools and throughout the Chicago area. While the connections, contexts and applications remained relevant, over time the numbers have changed. Gas is no longer $1.25/ gallon, salaries and the minimum wage have risen, and inflation has come down (at least for a while). The Great Economic Downturn of 2007 and the recession that followed raised new issues about the gap between the haves and the have-nots, and while there is progress in some areas such as equal pay for men and women, there are still gaps illustrating social injustices based on gender and ethnicity. Nowadays, it seems that in our political debates, we hear about the middle class and the upper class, but less about the poor.

This revision is an attempt by three mathematics teachers at Saint Ignatius College Prep (Chicago) to update and revise some of the original lessons from *Math for a Change* as well as to add some new lessons. For example, *Water – A Global Crises* is updated with 2012 statistics. We hope that new lessons, like *Decision and Justice in the Court System*, are more relevant and useful today with many students enrolled in AP Statistics. In our Table of Contents, we have suggested which math class the lesson might be used in, as well as the main mathematical skills and concepts. However, these are just suggestions. Many of the lessons can be used at any level.

The three of us who revised, edited, and wrote the lessons contained here would like to thank Laurie Jordan and the Mathematics Teachers' Association of Chicago and Vicinity for allowing us to update the original lessons as well as Dr. Cathy Karl, the principal at Saint Ignatius College Prep, for supporting us with ` summer curriculum funds. It has been a great experience for us to go back and reread the original lessons written by Fr. Thul and Mr. Mistrik as we selected topics. We hope that the revision has done them justice, and that you find these updates and new lessons useful and educational in your mathematics classrooms.

Bill Carroll, Cheryl Colyer, Sandie Bulmann

Saint Ignatius College Prep

1. Water – A Global Crisis

Some facts about water: (according to a WaterAid Report 2012 update)

- 78.3 million people in the world do not have access to safe water. This is roughly one hundredth of the world's population.
- 2.5 billion people in the world do not have access to adequate sanitation, almost two fifths of the world's population
- Around 700,000 children die every year from diarrhea caused by unsafe water and poor sanitation – that's almost 2000 children a day.
- Hand-washing could reduce the risk of diarrhea by nearly 50% per year.
- Women in Africa and Asia often carry water on their heads weighing 20 kg, the same as the average airport luggage allowance.
- 1.1 billion people live more than a kilometer from their water source and use just five liters of unsafe water a day. The average person in the USA uses over 570 liters of water a day.
- Water in Accra, Ghana costs three times as much as in New York.
- For every $1 invested in water and sanitation, an average of $8 is returned in increased productivity.

Please answer the following questions:

1. Approximately what is the world's population?
2. How many children's lives would be saved per year if water was accessible for proper hand washing? Per day?
3. Assuming a family who lives a kilometer from their water source has to refresh their water supply at least once a day, how many miles would they have to walk in a week?
4. Compare the water usage of the people living more than a kilometer from their water source with usage in the USA.
5. If the average African or Asian woman weighs 120 pounds, compare the weight of the water they often carry to their total weight.
6. If the average water bill in New York is $30 a month, what would be the cost in Ghana?
7. If just $25 were invested in global water and sanitation improvement, what would be the total return in increased productivity?
8. Water is a precious resource. What are two things you might do to help make water available in the future and for those who lack clean water now? Be specific.

Water – A Global Crisis

To the teacher:

The mathematical skills covered in this lesson include:

- Applying basic arithmetic operations
- Using ratios and percents
- Making metric conversions

Answers:

1. 78.3 million people equals one-hundredth of population, so total population would be 7.8 billion**.** [Or using the information in the second bullet, 2.5 billion equals two fifths of population so total population would be 6.25 billion.]
2. 50% of 700,000 equals 350,000 per year or 1000 per day.
3. 1 kilometer = .621 miles so total distance would be .621(7)2 = 8.7 miles
4. 5 liters versus 570 liters or .00877%. Or Americans use 114 times as much.
5. 1 kg = 2.2 lb. so 120 lbs. = 54.5 kg compared to water weighing 20 kg equals 1/6 or 37% of body weight
6. $30 times 3 would be $90
7. $25 times 8 would be $200.

2. Traveling to Foreign Countries

When traveling to a foreign country you need to obtain some of the local currency. You may do this before you leave at a foreign currency exchange office, i.e. the American Express office in downtown Chicago, or you may exchange your dollars once you arrive in the country you are traveling to. It is good to do your research and be cautious so that they do not take advantage of you. Last summer I traveled to Spain. I did my research and exchanged my money before leaving the U.S. However I was curious what the rates were once I arrived there. I found an exchange in the airport offering the following: "€0.78 = $1.00" (0.78 Euros = $1.00 American). They also listed "€1.00 = $1.17", which meant that one Euro would buy 1.17 dollars. So there were two rates of exchange, one for those entering the country who needed to buy Euros, the other for travelers leaving the country who had to get rid of foreign currency before leaving the country. So in addition to being wary about slick money changers, if I needed to exchange more money, I wondered if I also had to be concerned about the fairness of the rates posted outside the currency exchange.

Please answer the following questions:

1. Were the rates posted set so that a person leaving Spain got paid a just amount in exchange for the money they cashed in? In order to answer this question intelligently, you will have to figure out how to calculate rates of currency exchange from the data given. If the currency exchange paid €0.78 for $1, at this same rate what would they pay for 1 euro?
2. If you were paid $1.17 for every euro, how many Euros would you receive for one dollar?
3. If you bought $100 worth of Euros and then sold them back the next day at the rates given above, how much money would you lose? Show your work
4. Why do you think that the exchange rates are not the same? Does this seem unfair to you or justified?

Traveling to Foreign Countries

To the teacher:

The mathematical skills covered in this lesson include:

- Solving equations with ratios
- Making conversions in monetary systems

The Injustice:

This problem is meant to be a help to students who will travel in foreign countries, and for that reason it might be put aside for the end of the school year, right before vacation. As far as matters of justice are concerned, there is no injustice here, although at first there may seem to be, and the problem is presented as a caution against too easily accusing another of injustice before "doing one's own homework," thinking things through.

Answers:

1. Using the proportion, €0.78/$1 = €1/$X, you should receive 1.28 for every Euro instead of $1.16 being paid.
2. Using proportions, €0.78/$1 = €1/$X, giving you €0.85 as opposed to the €0.78.
3. At the exchange rates given, you would receive €78 for the $100. When you sold them back, you would receive $91.26 (€1/$1.17 = €78/$X) for a loss of $8.74.
4. One might at first expect the traveler to be paid $1.28 in exchange for every Euro, instead of $1.17 - the same amount despite the order of the transaction. But upon reflection one can recognize the right and the need banks and exchange agencies have to charge for their services if they are to stay in business. It's like them charging and paying interest. For instance, the historical average interest rate a bank was charging was 7.5%, but they were paying only an average of only 1% interest on savings that were deposited with them. So yes, it is necessary to have two different rates of exchange.

3. The United States Court of Justice

According to the Chicago Tribune dated June 29, 2012, Douglas Kelley, a 27 year old Batavia, Illinois man, carrying a gun, held up two restaurants and a motel and when caught was sentenced to 20 years in prison. The robberies netted him about $1300.

A few weeks later on July 20, 2012, according to the Chicago Tribune and the Sun-Times, Stuart Levine, 66 years old and a former political insider, allegedly masterminded a variety of illegal backroom deals. The crimes he actually admitted to were mail fraud, money laundering, and scheming to squeeze millions of dollars from firms seeking state business. For these crimes he was sentenced to about 5 ½ years in prison. He received a reduced sentence for these charges because he entered a plea agreement with prosecutors in return for cooperating as a witness in the corruption trial of other government officials. He also admitted to stealing $6 million from a charity, $2 million from the estate of a dead man's children, and trying to take part in a $1.5 million kickback bribery scheme, but he was not tried or sentenced for these crimes.

Please answer the following questions:

1. In Mr. Kelley's case how much money is represented for each year of his sentence?
2. In Mr. Levine's case, given only the crimes he admitted to, how much money is represented for each year of his sentence?
3. How much did cooperating in a government corruption trial save Mr. Levine in prison time assuming his sentence of 5.5 years was reduced by two thirds?
4. Assuming this reduction for his cooperation in the other trial, what would have been the sentence and amount per year for the additional crimes?
5. What should Mr. Levine's sentence have been if the ratio of amount stolen to sentence time were equal to Mr. Kelley?
6. What would Mr. Kelley's sentence have been if it were equivalent to Mr. Levine's sentence?
7. Mr. Kelley used a gun while Mr. Levine's crimes involved theft but there was no weapon involved. How might this have affected the sentence?
8. Do you think the sentences given to these two men were fair?

The United States Court of Justice

To the teacher:

The mathematical skills covered in this lesson include:

- Basic arithmetic skills
- Money and time conversions
- Ratio and proportion

Answers:

1. \$1300/20 years = \$65 per year
2. \$9.5 million/5.5 years = \$1.7 million per year
3. Sentence would have been 8.25 years
4. \$9.5 million/8.25 years = \$1.15 million per year
5. At \$65 per year, his sentence should have been \$9.5 million/\$65 = 146.2 years
6. At \$1.7 million per year, his sentence would have been \$1500/\$1.7 million = .32 days = 7.7 hours
7. Answers will vary
8. Answers will vary

4. Inhumane Working Conditions

According to a National Labor Committee Report (February 2009), workers are forced to labor under horrific working conditions in a factory in Dongguan City, China. Consider the following:

- Overtime is mandatory resulting in 12-hour shifts seven days a week with an average of only two days off a month.
- Anyone taking Sunday off is docked two and a half day's pay.
- The base pay is 64 cents an hour and after deductions for primitive room and board take-home wages are 41 cents an hour.
- Workers average up to 81 hours a week on site for a 74 hour workweek including 34 hours of overtime, 318% above China's legal limit.
- Workers are docked one and a half days pay for arriving over one hour late and lose three days pay for leaving their workstation without permission.

Please answer the following questions:

1. How many hours per week does a person have to work given these requirements? Per month?
2. Compare the number of hours per week required by these Chinese workers to what is a typical week in the United States.
3. What would a worker make in wages before deductions for one week's work? After deductions?
4. Compare this to what a typical worker would make here in the United States. To do this comparison you could look up the median income in the United States in 2012. You can also compute income assuming a US worker works 40 hours per week at a minimum wage of $8.25 per hour.
5. What would the weekly pay be if a person was one hour late?
6. What would the weekly pay be if the worker left his workstation without permission?
7. Why are these workers taken advantage of by the factory? What can you do about it?

Inhumane Working Conditions

To the teacher:

The mathematical skills covered in this lesson include:

- Solving multi-step arithmetic problems involving rates and ratios.

Answers:

1. 12(7) = 84 hours per week, 12(30) – 12(2 days off per month) = 336 hours per month
2. 84 vs. 40 equals about twice as many hours
3. 0.64(84) = $53.76, 0.41(84) = $34.44 per week
4. If a person makes $8.25/hour, the minimum wage in Illinois, then the weekly salary at 40 hours is about $330. For a family, this is below the poverty level, but nearly 10 times the weekly salary of a working in this factory in China. For the U.S., the median family income is above $40,000.
5. 12(7-1.5)(0.64) = $42.24
6. 12(7-3)(0.64) = $30.72
7. There are probably no other jobs for them, and the government may not regulate the workplace as in the U.S. Most likely they have no unions to represent and protect them. As a consumer, we can try to be aware of whether products we buy are from companies that treat their employees fairly rather than exploiting them.

5. Hunger on a Massive Scale

According to an article in the Los Angeles Times, (July 2012), nearly one billion people are malnourished, and a child dies of hunger every 11 seconds. By 2050, farmers would have to double crop production to meet demand.

This situation is particularly dire in Kenya where the poverty rate approaches 43%. The table below gives the poverty rate and other statistics for Kenya.

Please answer the following questions:

Country pop.: **42.7 million**

Fertility rate: **4.80**	Median age: **18.5**
In poverty: **43.4%**	Literacy rate: **73.6%**

Poverty is population living below $1.25 a day
Sources: United Nations, World Bank

1. How many children die each day of hunger worldwide?
2. If the world population is now 7,030,846,000, what percent are currently malnourished worldwide?
3. Given the data from Kenya, how many people are living in poverty right now?
4. If the poverty level is set at $1.25 a day, how much would a family income be for a year? Compare that amount to what people make in the U.S.
5. The fertility rate in the United States in 2011 was 2.06 children per woman. This number represents the average number of children born per mother. Compare this rate with the rate in Kenya.
6. Countries with high fertility rates, like Kenya, generally have higher poverty rates than countries with lower fertility rates? Why do you think this is?
7. How many people in Kenya do not know how to read? How do you think this compares to the U.S. How does this affect the poverty level?

Hunger on a Massive Scale

To the teacher:

The mathematical skills covered in this lesson include:

- Using basic arithmetic operations to compute and compare statistics;
- Using and interpreting statistics from a table.

Answers:

1. With about 86,400 seconds in a day/11 seconds = about 7,900 children
2. 1 billion/7.03 billion = 14.2%
3. 43.4% of 42.7 million = 18.5 million
4. $455 per year, median family income in the United States is around $40,000. Another comparison is to the minimum wage in the U.S. In 2012, the Illinois minimum wage was $ 8.25 per hour. Assuming a 40 hour work week, this is just over $17 thousand per year. For a family of 4, this is below the poverty level, but 37 times the income of a family making $1.25 a day.
5. 4.8 vs. 2.06 = 2.3 times as great
6. Generally countries with more children born (higher fertility rates) will have lower income than those with fewer children per family. In Kenya, the average family has more than double the number of children in the average U.S. family. Higher fertility rates are often correlated to greater poverty, higher infant mortality, and lower educational attainment.
7. 26.4% of 42.7 million = 11.3 million. The literacy rate depends on the definition of literacy, but estimates for the U.S. are around 90% literacy.
   Low literacy, poor educational opportunities, and poverty often go together.

6. The Effects of Inflation

Would you rather live in the America of 1950 or the America of 2012? Below are some facts from an article entitled The Economic Collapse, dated April 2012.

Cost of selected goods	1950	2012
Gasoline/gallon	27 cents	$3.69
First-class stamp	3 cents	45 cents
Bread	18 cents	$1.95
Television	$250	$300
Car	$1500	$12000
Cost of housing	22% of income	43% of income
Average income	$5000	$40000
Poverty level	$3016	$23200
Number living in poverty		46.2 million

What was the percentage increase for gas from 1950 to 2012? To find the percentage increase, R, you can solve the equation *R = (price in 2012 – price in 1950) divided by price in 1950*. For gasoline, R = (3.69 - .27)/.27 = 12.67 = 1267%

Please answer the following questions:

1. Compute the percentage increase in prices for the next 4 items listed above. Which item cost was least effected by inflation?
2. Using a line graph estimate what the cost of a car would have been the year you were born.
3. Compare the increase in housing costs to the increase in prices.
4. What was the average housing cost per year given average income? Per month?
5. Using a line graph estimate what the monthly cost of housing would have been in the year you were born.
6. What percentage of average income is the housing cost for people living at the poverty level in 2012? How does this compare to 1950?
7. Using your answer in #6, explain the effect of inflation on the 46.2 million people living in poverty? Why do you think the cutoff for the poverty level has increased eightfold in these 62 years?

The Effects of Inflation

To the teacher:

The mathematical skills covered in this lesson include:

- Finding percent and percentage increase
- Using linear equations and slope to find and apply rates of change

Answers:

1. Stamp = 1400%, bread = 983%, television = 20%, car = 700%. The cost of a television has increased less than the other items.
2. Let x equal the number of years since 1950 and y equal the cost. The ordered pairs would be (62, 12000) and (0, 1500). The slope of the line would be (12000 – 1500)/62 = 169. The equation of the line would be y = 169x + 1500. If a student was born in 1996, then x = 46 and y = $9274.
3. (43-22)/22 = 95%, in other words housing has increased around 100% or doubled. While this is a lower rate, it takes a larger share of people income.
4. 22% of $5000 = $1100, about $92 per month in 1950, 43% of 40000 = $17200, $1433 per month in 2012.
5. Let x = the year since 1950 and y = the monthly costs from Question 4 above. The ordered pairs would be (0,92) and (62,1433). Slope would be (1433-92)/62 = 21.6. The equation would be y = 21.6x + 92. If x = 46 then y = $1087 per month.
6. In 1950, the average cost of housing was 1100. Using the top of the poverty level, in 1950 the percentage of income was 1100/3016 =36%, using the high end of the poverty cutoff. In 2012, 17200/23200 = 74%. People are spending more of their income on housing. This would especially affect those at the lower end with less money. Following the economic downturn of 2008, large numbers of people lost their homes, and housing advocates say that the number of homeless people has greatly increased.
7. Answers will vary. It is necessary to adjust the poverty level to keep up with inflation.

7. The Point-O-Eight Law

Stopping after work for a few beers is a common occurrence for many people. However when those beers raise the Blood Alcohol Content (BAC) too high, the lives of other people can be placed on the line.

Too much human life is being lost every year because of drunk drivers. It can happen to anyone at any given time. Recently a family of five was returning late at night from a family reunion. Speeding around a curve from the opposite direction was somebody “under the influence”. The sudden flash of headlights startled the intoxicated driver and he plowed right into the family’s mini-van. On impact the entire family was killed. The drunken driver survived with only a broken arm.

In 2009, 911 people died on Illinois roads because of drinking and driving, and they’re involved in more than 40% of all high way fatalities. If a person’s Blood Alcohol Content (BAC) is below .05, there is “possible impairment” of the driver’s ability to concentrate and control speed. If the BAC is between .05 and .08, there is “some impairment”. If the BAC rises to .08 or higher, there is “definite impairment”, with motor skill reaction times significantly increased so that he or she is three times more likely to be involved in crashes and 11 times more likely to be involved in fatal crashes. Forty five states, the District of Columbia, and Puerto Rico have adopted the “Point-O-Eight” law, meaning that it is illegal to drive with a BAC >.08.

On July 2, 1997 after six years of trying, Illinois enacted the “Point-O-Eight” law. Since then, Illinois has seen an increase in the number of lives saved. A study of the effectiveness of a .08 BAC law implemented in Illinois in 1997, found that the .08 BAC law was associated with a 13.7 percent decline in the number of drunk drivers involved in fatal crashes. The reduction included drivers at both high and low BAC levels. This is significant because critics of .08 BAC laws have often claimed that these laws do not affect the behavior of high BAC drivers.

But there are those who argue against the “Point-O-Eight Law”. They claim that driving with a BAC under .10 is not the problem, but it’s the chronic drunken driver who is responsible for the majority of alcohol-related accidents. In 1993 only 8.5% of Illinois’ fatal accidents involved drivers with a BAC of <.10. 36.7% involved drivers whose BAC was over .10 and 54.8% did not involve alcohol at all. They claim the “Point-O-Eight Law” unfairly impacts the merely social drinkers who have a right to relax in a suitable way. It targets the wrong people and fails to curb the culprits. The illegal BAC level should have been left at .10.

What do you think? Can mere social drinking result in a BAC of "Point-Ten"? Or does this take a bit more serious drinking?

Let's Math-Investigate It

1. Below find a table that gives the BAC for people of different weights when 0.5 ounces of alcohol have been consumed. Use the math tool of your own choice to complete the table. You can carefully graph the points of the table, drawing a smooth curve through them, and continuing the curve up to 220 lb. Or you can plot the table points on a graphing calculator and choose a regression model that will give you the desired BAC values.

Lbs Weight	60	80	100	120	140	160	180	200	220
BAC for 0.5 oz	.05	.044	.038	.033	.028				

2. For any given weight level the BAC is directly proportional to the number of ounces of alcohol imbibed. Calculate the number of ounces of alcohol that will raise the BAC of a person weighing 180 lbs. to "Point-O-Eight".
3. A serving of beer is 12 ounces and contains 4.5% alcohol. Figure out the number of beers consumed within one hour that will raise the BAC of a person weighing 180 lbs. to "Point-Ten".
4. Based on your answer in #3, is the drinking done by the person merely social drinking, or is it a type of drinking that's a bit more serious?
5. Are you an advocate of the "Point-O-Eight Law" or an opponent? State your reasons. Jot down what you could do to promote the side you believe in.
6. Does a person need to be legally drunk to drive impaired? Could their driving be impaired if they drank less beer? If so, what are some of the potential consequences?

Government Warning (Found on beer can labels)

1. According to the surgeon general, women should not drink alcoholic beverages during pregnancy because of the risk of birth defects.
2. Consumption of alcoholic beverage impairs your ability to drive a car or operate machinery, and may cause health problems.

The Point-O-Eight Law

To the teacher:

This is a good problem to coordinate with sophomore drivers' ed class, any accidents or near accidents that may be in the news or around school, and with the senior prom.
The mathematical skills covered in this lesson include:

- Using linear rate of change or a regression equation to make prediction
- Solving and using direct variation

This problem tests the student's understanding of "direct proportion", calls for careful graphing and/or for finding regression models on a graphing calculator. Several procedures are to be gathered together into the final solution. It should ordinarily be reserved for second semester high school sophomores and for upperclassmen.

It would be interesting to have some students in class solve question #1 by extrapolating the curve as suggested, and others solve it by a regression model. Students have to be encouraged to get away from merely pushing "magic buttons" on calculators without having any idea of the math behind those buttons.

Answers:

1. The desired BAC values are:

Lbs Weight	60	80	100	120	140	160	180	200	220
BAC for 0.5 oz	.05	.044	.038	.033	.028	**.022**	**.017**	**.011**	**.006**

   It is recommended to use a graphing calculator to perform a regression model to obtain the equation $y = -3628x + 240$. Using this allows the student to answer the remaining values above.
2. 2.35 ounces. Use proportions for a 180 pound person to calculate. 0.017/0.5 = .08/x therefore, x = 2.35 oz.
3. 5.4 beers. This is a two step process:
   a. Use proportions with the above information and compare to .10 ounces.
      Therefore we get: $\frac{.5}{.017} = \frac{x}{.10}$ Therefore x = 2.94
4. The person needs to consume 2.94 ounces of beer, and a beer contains .54 ounces (12 ounce can X .045 alcohol = .54). Therefore $\frac{2.94}{.54} = 5.4$
5. 5.4 beers in an hour is pretty serious drinking. Answers will vary.
6. Answers will vary.

7. No, their driving could be impaired if they drink less. Serious injury, possible death, and severe legal issues could result.

Additional Driving Safety Facts (http://www.dot.state.il.us/trafficsafety/factsheet.html)

The Magnitude of the Problem

- Nearly three out of every ten Americans will be involved in an alcohol-related traffic crash in their lifetime. • Each year, about 8 percent of all police-reported motor vehicle crashes are alcohol-related. In Illinois during 2002, 51,649 people were arrested for DUI.
- The proportion of fatal crashes that are alcohol related is approximately three times greater at night than during the day.
- Each year, about 310,000 people suffer injuries in alcohol-related traffic crashes, an average of one person injured approximately every 2 minutes.

Blood Alcohol Concentration (BAC)

- A blood alcohol concentration (BAC) of .08 or greater is the level at which a driver is considered legally intoxicated in Illinois .
- A driver can also be arrested and prosecuted for Driving Under the Influence (DUI) with a BAC in excess of .05 but less than .08.
- In 2002, 45.83 percent of fatally injured drivers who were tested for a BAC level were found to have been drinking, and 39.54 percent had a BAC of .08 or greater.

Drunk Driving and Young People

- Although 16-24 year olds comprise only 15.52 percent of the licensed drivers in the state, they are involved in 38.85 percent of all fatal alcohol-related crashes.
- In 2002, nearly 32 percent of the fatally injured teenaged drivers (age 16-19) were legally intoxicated.
- 224 young adult drivers between 16 and 24 years old were killed in fatal crashes in 2002. Of these, 106 had a BAC level of .08 or greater.
- Nearly 33 percent of the fatally injured teenage drivers (age 16-19) were drinking prior to their crash.
- Of the 396 drivers involved in fatal crashes in 2002 and found to be legally intoxicated, 35.10 percent were between 16 and 24 years of age.
- Almost 37 percent of the fatally injured drivers under age 21 who were tested for BAC were drinking prior to their crash. 39.66 percent were at .08 BAC or greater.
- In Illinois , in 2002, 83 children under the age of 16 were killed in motor vehicle crashes.
- Illinois ' zero tolerance law became effective January 1, 1995 . Each year there are approximately 3,000 zero tolerance violations recorded.

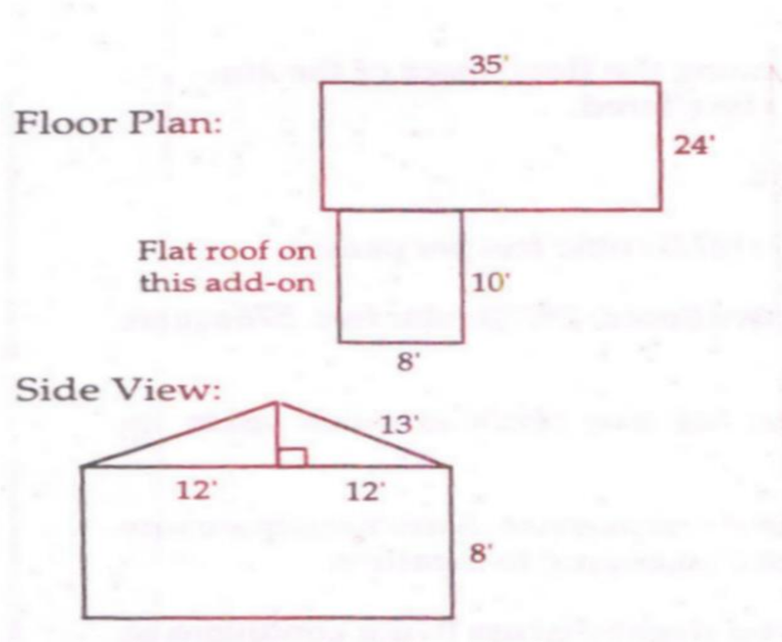
Safety Belts and Alcohol

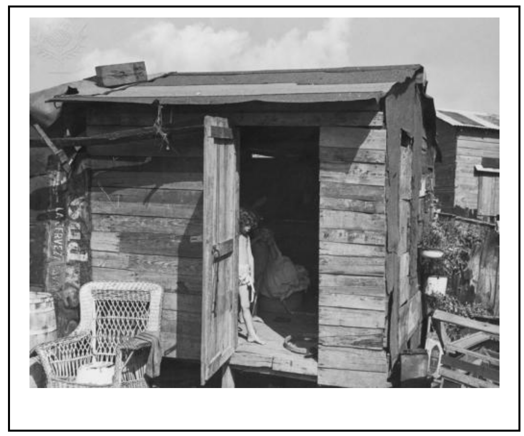
- Safety belts were used by approximately 12.3 percent of fatally injured intoxicated (BAC> .08) drivers as compared to 36.7 percent of sober drivers killed in crashes.
- Drivers involved in fatal crashes who have been drinking use safety belts at a substantially lower rate than sober drivers.

8. Overcrowded Living Conditions

Many Americans complain when they have to share a bathroom with another family member, or even worse, with overnight guests. But there are other Americans who are just glad to have a roof over their heads. Consider the Green family from Wallins, Kentucky, which is located in the heart of Appalachia. Their house is typical of many shacks in the mountains of Eastern Kentucky. Mr. and Mrs. Green live with their six children in the house pictured below. (The measurements are given in feet.)

Figure 1: Floor plan for Green Family Home





Now consider a newly constructed house typical of recent single-family homes in an affluent Chicago neighborhood. This is a two-story home with a finished basement (family room). The measurements and other statistics for this home are given below. A family of 4 lives there.

Table 1: Family for Chicago Home

Use	Single Family
Exterior Construction	Frame
Full Baths	3
Half Baths	1
Basement	Full and Rec Room
Central Air	Yes
Number of Fireplaces	3
Garage Size/Type	2 car detached
Building Square Footage	2,280 (first and second floors included)

Note: Area of basement/family room is not included in the building square footage in table. Assume the basement/family room is identical in area to the other floors of the house.

Please answer the following questions

1. Find the area of the Greens' house using the correct unit.

2. Find the volume of the Green's house, including the attic using the correct unit.

3. Compute the area per person for the Green's home and for the family of 4 in the Chicago home. Make sure you add in the finished basement/family room, and use the correct unit of comparison.

4. Compute the volume for the Chicago home assuming it has 8 foot ceilings.

5. Compare these 2 housing situations. Aside from area, what are some other aspects of the living conditions on which they differ? In your opinion, do the Greens live in adequate housing? Would you want to live in their house?

6. Measure the house or apartment in which you live. Draw a scale model diagram and calculate its area. Include your scale. Determine the area per person and compare it to the two situations above.

Overcrowded Living Conditions

To the teacher:

The mathematical skills covered in this lesson include:

- Computing area, volume, and using the correct unit (square and cubic units;
- Making and interpreting ratio comparisons
- Make measurements to create a scale model diagram and compute area of polygons.

Answers:

1. The area of the first floor is 920. If the attic is included, the area is 1760 square feet.

2. 7360 + 2100 = 9460 cubic feet.

3. Including the attic, there are 220 square feet per person in the Green's home. In the Chicago home, there are 3420 square feet, including the family room. This gives 855 square feet per person. This is nearly 4 times as much space per person in Chicago compared to the Green's home. If the basement family room is not included, there is still 570 feet per square feet per person, more than twice the Green's situation.

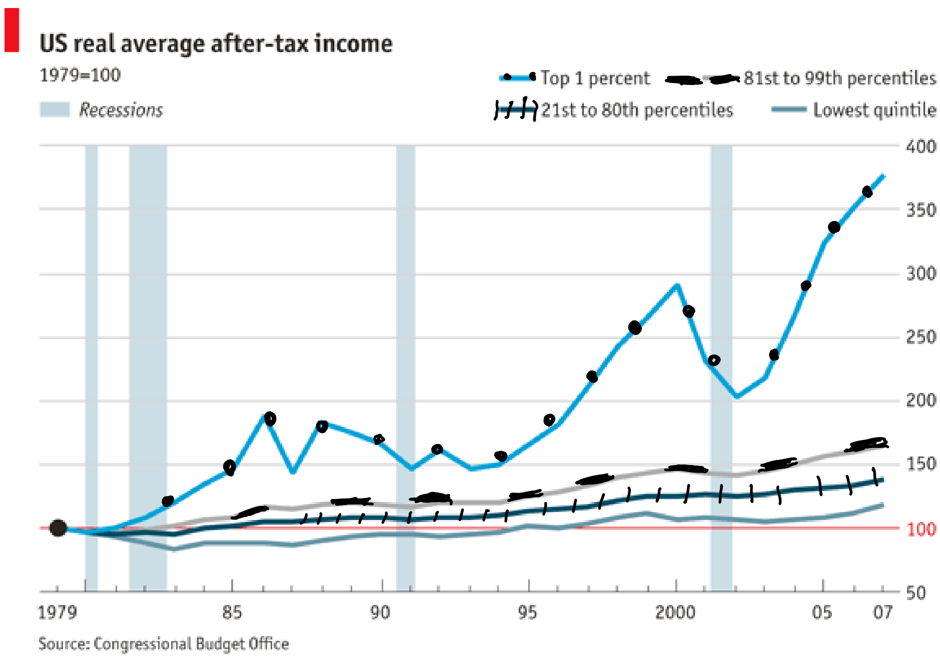
4. Assuming the rooms are all rectangular solids, the volume is area * height = 27,360 cubic feet including the family room. Without the family room, it is 18,240 cubic feet. Check to see that students can write and read these units correctly. A common mistake is to read them as "feet cubed" rather than "cubic feet."

5. The Green's home is quite humble compared to the affluent Chicago home. The Greens likely have only one bathroom, compared to 3.5 for the Chicago family. The Chicago home has central air conditioning, 3 fire places, and a two-car garage, amenities the Green family does not have. Many things that were once luxuries or niceties – like air conditioning, are now considered necessities by many people.

6. This will be a nice activity that involves the students in measuring, drawing diagrams, and computing area. These diagrams and write-ups can be interesting bulletin board displays – but watch out for inequity in your own classroom that might embarrass some students.

9. The Rich Get Richer: Their Piece of the Pie

We cannot expect everyone to have the same income. Some people have more education, some work more hours, some lose their jobs, especially during hard times like the world-wide economic downturn and recession that began at the end of 2007. While we might not expect everyone to get an equal "piece of the pie," we would hope that everyone gets a reasonable share. In one of the world's richest countries, where we claim everyone has a chance, large gaps in inequity should not be acceptable.



Unfortunately, it seems that the gap between the richest and the poorest Americans is growing, and while everyone suffered during the economic downturn, it is the poorest Americans who suffered the most in many ways. Graph 1 showed the income of Americans of different income levels by percentile. You know percentiles from your standardized test scores – if you are at the 99th percentile, you scored above 99% of those who took the test. You are in the top 1% of testers. This graph from *The Economist* shows that the amount of wealth controlled by the top 1% of Americans has been increasing. Unfortunately, the graph only goes up to 2007, just before the economic downturn.

More recent data from the U.S. Census Bureau divides American households into quintiles by income. Quintiles are percentiles that divide the population into equal fifths. For example, if your family is in the 1st quintile, you earn less than 80% of American; you are in the bottom 20%. The table below shows the income levels for households and the percent of the nation's wealth that they hold.

Table 1: U.S. Income Distribution by Quintile

Quintile	Income	Percent of nation's wealth
1	$20,000 or lower	3.5
2	$20,000 – 38,043	8.5
3	$38,044 – 61735	14.6
4	$61,736 – 100,065	23.4
5	$100,066 and higher	50.2

Source: U.S. Census Bureau, 2010

While economic downturns affect all Americans, it seems like those at the bottom are hit harder. According to the U.S. Census Bureau, the income ratio of earners at the 90th percentile to those at the 10th percentile increased between 1999 and 2010, from 10.5 to 11.7. The gap between the richest 10% and the poorest 10% is greater than before the downturn. In 2012, about 1/6 of Americans live in poverty.

Please answer the following questions:

1. The time plot for real average after-tax income goes through 2007. What does it show about the distribution of wealth from 1979 to 2007?
2. A major economic downturn and recession began at the end of 2007. Which group do you think were most affected by this downturn? Why?
3. Make a circle graph dividing the circle into areas representing the distribution of wealth among the quintiles (Table 1). What does it show you?
4. What central angle would you use in your circle graph for the 1st quartile? For the 5th quartile? Show your work.
5. The U.S. Census Bureau reports by household. Why is it incorrect to say that 20% of Americans are at the lowest quintile and earn $20,000 or lower?
6. One way of looking at the disparity in family incomes is, "People who work hard should be rewarded for their work." Another way is, "A large gap between the poor and the rich reveals the inequity in our country." We admire the rich and yet complain about the top 1%. What do you think about the income gap?
7. Families at the bottom struggle to make ends meet. Food, shelter, clothing, health care, and other essentials might be cut to make ends meet. Children in poor families lack health care, educational opportunities, and other things we might take for granted. What are some ways you can help the poorest Americans?

Their Piece of the Pie

To the teacher:

The mathematical skills covered in this lesson include:

- Reading and interpreting a line plot from ordered pairs.
- Making a circle graph.
- Computing the measure of an angle in a circle graph.
- Understanding quintiles, percentiles, and interpreting data.

Answers:

1. 1. The time plot is a little difficult to interpret because it does not break people down into equal percentiles (quartiles or quintiles) by income. However it does show that the top 1% of the population makes significantly greater income, and that their share, relative to the rest of the population, has increased over time. A numeric comparison would be nice, e.g., the richest 1% has nearly quadrupled their earnings over these years while the earnings of lowest quintile have not even increased by 50%.

2. The richest 1% did show the biggest decrease in time of recession, but still controlled most of the nation's wealth. As the article states, the gap between the richest and poorest has grown over these 28 years.

1. The graph should show that the top 20% has more than 50% of the wealth. The bottom 20% has a very thin slice of the pie, about 1/14 of the top 20%.

2. Lowest quintile: 3.5% of 360 = 12.6 degrees. Highest quintile: 180.72 degrees.

3. The unit of analysis for the Census Bureau is the family. The number of children who fall into the lowest quintile is greater than 20%.

4. Student's answers will vary. It is not unreasonable that some will have more than others. Many factors affect success. But the graph and the table show disparity that seems greatly skewed, in one of the richest nations, more than 15% of the people live in poverty.

5. Answers will vary. Economic aid, donated food and clothing, and volunteering at charities are all good ways of helping the poor. Improving the educational level of all is a more long term situation. Perhaps some students might consider teaching in poor urban and rural schools to help raise the educational level and potential for children living in poverty.

10. Just Do It!: Using a Mission Collection to Investigate Association

The situation in Iraq, Afghanistan, Syria, and Sudan continues to be a daily challenge for its people. Every day on television we see the stories that come out of the war, including child marriages, girls being poisoned or killed for attending school, wounded citizens, and other injustices. The people need help. Many agencies collect money, food and clothing to be sent over there. Imagine that one of those agencies elicited your help. What would you do?

When they hear appeals such as this, some people react in somewhat the following manner: "Oh, social problems are too massive ever to be solved. There's nothing one can do, really. After all, it's really their problem. They'll have to solve it. We have enough problems of our own."

The reactions of other people can be very different. They feel really bad for human beings suffering so much. They refuse to give in and to say there's no way to help. They ask themselves sincerely what can be done; they may make a few inquiries of other people; they might read a blog; in the end they usually come up with something, and they do it.

Now there are hundreds of factors that influence the way a person reacts to this or that injustice or social problem. In this project we will look into just one of those possible factors as it might show itself in just one given situation. The given situation will by your school's last mission collection, presumably taken up in their homerooms. The factor to be tested will be each student's financial means: we will try to see whether the amount of money a person has at his or her disposal has anything to do with how much that person contributes to the mission collection.

Here's what to do:

1. With the help of your teacher you are to form teams. Each team is to collect data points from the homeroom assigned to it.
2. The data point is to contain two things: 1) the average number of dollars per week a student has at his/her disposal, and 2) the number of dollars she/he contributed to the collection.
3. Each team member will get two such data points. Honesty is essential.
4. Then by team or by class, plot the points on a pair of coordinate axes, with the number of disposable dollars along the x-axis and the number of dollars contributed along the y-axis. This coordinate system is called a "scatter gram" of the data points.
5. Finally, your teacher will tell you how straight lines are fitted to scattergrams. Find your fitted line and report its slope to your class. This slope is the answer you have been looking for. The greater the slope, the truer will it be that the amount of money a person has at his or her disposal determines how much she/he contributes to mission collections. Note: your teacher may want to construct a scatter plot for the whole class.
6. Say you find an association. What factors might make the validity inaccurate?

Just Do It!

To the teacher:

The mathematical skills covered in this lesson include:
Making scatter plots and regression lines, neat topics that can easily be overlooked in a math course. Algebra 1 students who have had slopes and the equations of straight lines can handle it, but at the same time, there is much that will be of interest to upper classmen as well. The time allowed for this project should be more than that of an ordinary homework assignment.

Answers:

1. Teams of four or five members would seem to be appropriate for this project. This will yield eight or ten data points, and that should give meaningful results. Students may find an association, linear or otherwise. Or it may be that there is no association.

2. The team is limited to working in one particular homeroom because in this way all the members of the class will have been exposed to the same motivations from the homeroom teacher, etc.

3. Students may need assistance setting up an appropriate graph and plotting the points. This is a good chance to informally assess these skills.

4. The reason the y-axis units are made greater is because the students' disposable money will presumably be considerably more than the money contributed to the collection.

5. The teacher can use the following option:
    - Have the students look up the procedure for fitted lines or for regression equations from the calculator for themselves in their Algebra 1 or Algebra 2 book. The teacher can also refer to the notes at the end of this exercise.

An alternate approach to this problem would be to make one giant scattergram for the whole class. In this case teams would only have to report data points, and the process of fitting a line to the data could be demonstrated in class.

SCATTER PLOTS
BEST-FIT LINES ON THE GRAPHING CALCULATOR

Provided by Erin Nolan

STEP 1: STAT, # 1 EDIT, Enter lists (L1, L2) with data. Make sure to press ENTER after your final data entry

STEP 2: 2nd Y = (STAT PLOT) , ENTER, make sure to turn on your plots- the ON should be highlighted,

Xlist: L1, Ylist: L2, (Now Plot 1 is highlighted in the top left)

STEP 3: ZOOM #9(ZoomStat)

STEP 4: STAT, right arrow to CALC, #4 LinReg (ax+b), ENTER, A: slope and b:y-intercept

STEP 5: Y =, enter your "lin reg" equation- don't forget the X. GRAPH

STEP 6: 2nd GRAPH (TABLE), use this to make predictions and find different values

**Use y= equation to make your predictions

**To clear your lists, you have to arrow to L1 press CLEAR and ENTER

11. Payday Loans (Exponential Growth)

When Joe Johnson's car broke down, he needed it fixed quickly to be able to get to work. Every day he commuted from his inner city neighborhood to the outer suburbs where many of the manufacturing jobs had moved. It was a long drive, inaccessible by bus, but at least it was a job that supported his family. The auto mechanic told him the repairs on his car would cost around $250, but Joe was tapped out. He no longer had a credit card, and most of his savings were needed for the rent and utilities for the month.

For a fast loan, Joe turned to a Quick Cash, a payday loan company. They would lend him $250 for two weeks, but he needed to bring in a copy of his recent paycheck and a personal check. The check was made out to Quick Cash dated two weeks from then. The cost for the two week loan was $15 for every hundred dollars loaned. This seemed like a lot to Joe, but the process was quick, and he really needed the car fixed, or he could lose his job. And he remembered that the annual percentage rate on his credit card had been around 15% so it didn't seem outrageous. Joe wrote his check for $287.50 to Quick Cash, due in two weeks, and immediately received $250 from them. He was off to work that afternoon.

When two weeks rolled around, Joe did not have the $287.50. One of his children had a dental emergency, the rent was due, and the family needed food on the table. The rising price of gas was eating into his paycheck too. At Quick Cash, he paid down $150 and asked them to roll the remaining amount over for another 2 weeks. He wrote a new check for $158.13 to Quick Cash, due in two weeks. This was for the remaining $137.50 plus the two week interest, again at $15 per $100 borrowed. This was less than the $287.50 he owed before, so he felt like he was making progress.

Please answer the following questions:

1. The two-week interest rate Joe paid was 15/100 or 15%. However, interest is usually reported as APR or annual percentage rate. What was the annual percentage rate Joe was paying? How does this compare to interest on bank loans, mortgages, and credit cards? (You might check with a parent, call the bank, or check rates online.)

2. If Joe paid off the loan at the end of the 4 weeks as described above, how much did he pay in interest? What percent of his original loan is this? What is the APR that he paid on his loan?

3. Sometimes people take a payday loan and miss payments. If you took a $250 payday loan at 15% every 2 weeks as Joe did and were unable to make any payment for a full year, what amount would you owe? You may use the equation

   $$\textit{New amount owed} = \textit{original amount} * 1.15^{t}$$

   where t is the number of two week periods since the loan.

4. Make a graph showing the amount due at the end of every 2-week period for about 6 months (24 weeks) if you took a $250 loan and were unable to make any payments. What type of mathematical model is shown here?

5. Sometimes places like Quick Cash are called predatory lenders because of the high interest they charge. Others say that while they are expensive, they fill a need for high-risk people who need money and have no other source. They are usually found in poorer neighborhoods. What do you think about this? What are some other good choices people like Joe Johnson might have when they are in a crunch for money?

Pay Day Loans

To the teacher:

The mathematical skills involved in this lesson include:

- Working with percentages and computing annual percentage rate (APR);
- Using the formula for compounded interest;
- Making a graph and recognizing it and the formula as describing exponential growth. Answers:

Answers:

1. At 15% for 2 weeks, this translates into a 390% APR: .15 * 52/2. However, if the loan is allowed to roll over, the APR increases due to compounded interest resulting in a much higher APR. Rates for banks and other lending institutions vary, but 3 to 6% is typical now, depending on the type of loan. Credit cards charge much more, but still they are below 20% APR, nowhere near the APR charged by payday loan agencies.

2. At the end of the first two-week period, Joe paid 150. At the end of the second two week period, he paid 158.13 for a total of $308.12 on his $250 loan. The interest he paid was 58.13/250 or 23.25%. However, as an APR, this is over 279% (23.25% per month for 12 months).

3. If nothing is paid off, an astounding $9,464.20 would be owed at the end of one year. This can be computed by 1.15^26 * $250.

4. The amount owed at the end of twelve 2-week periods is shown below. A graph should clearly show this as an exponential model. Amounts are rounded to the nearest dollar.

2- week periods	1	2	3	4	5	6	7	8	9	10	11	12
Amount owed	287	331	380	437	503	578	665	265	879	1011	1163	1137

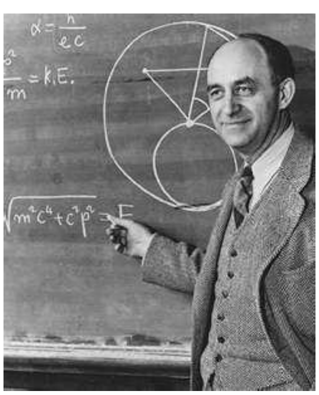
A graph, or an analysis of the table, clearly shows exponential growth.

7. Payday loan agencies do play a role when poor people need quick money. However, with such large interest rates, it is easy for people to fall further and further behind as the debt grows exponentially. In some cases, people may have no other resource, but they should at least be educated about the risks they are taking when borrowing from a payday loan agency. Generally a person would be much better off taking a loan from a family member or a friend.

12. Hunger and Poverty in the United States – Part 1: A Fermi Problem

What percent of people in the United States live in poverty? Altogether how many American children live in poverty? Sure you could look it up on the internet or ask the librarian to help you research the question. But can you come up with a reasonable answer reasoning from just some basic facts?

This type of problem is called a Fermi Problem, after Enrico Fermi, famed physicist from the University of Chicago. He would ask his students to answer a question given only a few facts and no books or resources. For example, one Fermi Problem is “How many piano tuners are there in Chicago?” Starting with the number of people in Chicago, the students would think about how many average-sized households this might be. Thinking about households they knew, they could estimate the percent that had pianos. With no more information than that, students could converge on an answer. Amazingly, students who reasoned well tend to come up with similar answers, and these generally reflect the true answer.



Enrico Fermi

Using this approach, see if you can estimate the number of children in the U.S. who live in poverty[1]. A child is anyone under the age of 18. Here are the facts you can use from the 2010 U.S. Census Bureau:

- Population of the United States: 308,745,538.
- In 2010, 11.7% of families living in poverty.
- The average size of a family/household: 3.14.

People living in poverty come from all parts of the U.S.: rural areas, cities, small towns, and even suburban areas. They include children, teens, adults, and the elderly.

Don’t cheat by trying to look on the internet. See how good your mathematical reasoning and numeracy skills are at making and using reasonable estimations. One tip – since you are making estimates, work with rounded numbers for example, you might use 300 million as the population of the U.S.

- [1] Definition of poverty: Annual income of $11,139 for a single person; $14,218 for a family of 2; $22,314 for a family of 4

Please answer the following questions:

1. How many children in the U.S. do you think live below the poverty line? Explain the numbers and the steps you used to come up with your estimates. How confident do you feel? Do you think you made an over- or underestimate?

2. Chicago has a population of about 2.7 million people. Based on your estimate, how many Chicago-sized cities would be filled by the number of poor children in the United States?

2. You were given 3 pieces of information. What are 2 other bits of information that would have helped you make a better estimate?

Hunger and Poverty in the United States: Part 1

To the teacher:

The mathematics skills covered in this lesson include:

- Numeracy skills: Using information to make and apply reasonable estimates;
- Thinking about numbers and percentages of people in poverty.

This lesson is preparation for the one that follows, "Hunger and Poverty in the United States: Part 2," by having students begin to think about poverty in America. But it also stands alone as a lesson on estimation, numeracy, and reasoning. Although estimation is an important skill (if you think about it, we probably estimate more often than we make exact calculations in real life applications of math), this skill is rarely developed in school. I remember once walking with a senior on our math team, one of our very top students, and asking her how many pigeons were flying in a group above us. She looked at me and said, "I have no idea how to answer that question."

It may well be that students in your class converge on answers of the same order of magnitude. This should not be surprising. Hopefully it will be in the same ballpark as the true answers. But the point of this lesson is to use numeracy skills -- like estimation -- and reasoning skills – like considering what facts might be helpful.

Students should be able to complete this activity in around 10 minutes. You might want them to work with a partner or even in small groups. At the end, a discussion of answers, and how students got them, would be useful. Or perhaps it might be a whole-class problem solving activity and discussion.

Answers:

1. According to the U.S. Census Bureau, the poverty rate for Americans in 2010 was 15.1%. This is about 46.2 million Americans. Children fare even worse than adults. In 2010, 20% of Americans under 18 years of age lived in poverty, up 1.3% from 2009. This is about 16.2 million children living in poverty.
   However, the purpose of this problem is not to arrive at an exact answer. It is to begin thinking about poverty and to exercise number sense.
2. The answer will depend on the students estimate. But given 16 million in poverty, the U.S. population of children in poverty is nearly 6 cities the size of Chicago.
3. Some things that might be helpful: What percent of the population is under 18? How many U.S. households live below the poverty line? What is the size of the average family?

Statistics used here are from the U.S. Census Bureau's report
Income, Poverty, and Health Insurance Coverage in the United States: 2010 (issued 2011).

13. Hunger and Poverty in the United States: Part 2

Four years ago, Sherry Murone got a late night call from the police. They were holding two boys who had broken into the community center of a local housing project, and they needed Murone's help. "What they found when they responded to a breaking-and-entering call really got to them – you could tell," recalled Sherry, head of the Second Harvest Food Bank in Savannah, Georgia.

It was two brothers, eight and ten years old. The little one was sitting in front of the open fridge, guzzling from a milk container. The older one was stuffing his pocket with fruit. When the police asked him why he had broken in, he just looked at them and said, "My brother was hungry." That, Murone knew, was the real crime. That night she vowed to help solve it. Sherry Murone founded Kid's Café, a chain of centers that serve nutritious meals to hungry American kids.

Poverty in the U.S. is, in fact, more widespread than most would believe. The recent economic collapse and recession that began in December, 2007, has worsened the situation for many families. In 2010, the federal government defined the poverty line as $ 22,314 for a family of four. In 2010, of the 305,688,000 people in the U.S., about 46.2 million Americans lived in poverty, an increase of 2.5% since this recession began. Of these poor, about 16.2 million were children under 18 years of age, 26.3 million were 18 to 64, and 3.5 million were above 64 years of age. The proportion of children in poverty increased more than the other groups[2].

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[2] The data in this report are from the 2011 Current Population Survey Annual Social and Economic Supplement (CPS ASEC) and were collected in the 50 states and the District of Columbia.

Please answer the following questions:

1. In order to make sense of these statistics, determine what percent of the U.S. population lives in poverty.

2. What percent of the poor are children under 18? Why is the poverty level so much higher among children? What can be done to alleviate the problem?

3. Use a Cartesian coordinate system to plot the following points and connect the points with line segments[3]:

Year	Millions of Poor	Year	Millions of Poor
2000	31.1	2006	36.5
2001	32.9	2007	37.3
2002	34.6	2008	39.8
2003	35.9	2009	43.6
2004	37.0	2010	46.2
2005	37.0		

4. Use your graph to predict the number of poor in 2015 if the current trend continues.

5. The regression equation for the 10 year period given is:

   Estimated millions of poor = 31.25 + 1.24 X *years after 2000.*

   For example, the value used to predict the millions of poor in 2011 would be 11.

   Use the regression equation to predict the millions of poor in 2015. How does this compare to your estimate in question 4? Explain any differences in the estimates.

6. What does the 1.24 in the regression equation mean? What does the 31.25 tell you?

7. Does it make sense to use regression to make a prediction for 2015? Explain.

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[3] Data on poverty comes from the records of the U.S. Census Bureau, http://www.census.gov/hhes/www/poverty/index.html

Hunger and Poverty in the United States: Part 2 (Continued)

To the teacher:

The mathematical skills covered in this lesson include:

- Graphing a time plot based on poverty rates;
- Using a regression equation to make a prediction;
- Thinking about the reasonableness of using linear regression to model a situation;
- Calculating with and using ratios and percentages.

Answers:

1. Based on these census estimates, 46.2 million / 305.688 million = 15.1% of the population lives in poverty as defined by the government.

2. 16.2/46.2 or about 35% of children live in poverty. When a mother is the sole support of a family, or the mother and father lose their jobs, the effect goes beyond them to their family. Families are more likely to fall into poverty than single people. Students' answers about alleviating the problem will vary.

3 and 4. While not completely linear, there is a general linear trend, although the number of poor did decrease in 2006. Since then, the increase has been steeper. The estimate will be based on the student's line of best fit. My estimate based on a graph of the ten-year period was about 45- 48 million people would be in poverty in 2015. But if the current trend continues, it will be much higher.

5. The prediction is about 49.8 million, but graphing the least square regression line with the scatter plots suggests that this is an underestimate.

6. This 1.24 in this equation predicts an increase of about 1,240,000 people into poverty every year. The 31.25 estimates the millions of people in poverty in the year 2000 based on the LSRL model.

7. There is the danger of extrapolating regression models beyond their range when we don't know what the trend is. This depends largely on the economic state of the U.S. and the world in the next few years.

14. Up In Smoke

Many teens have experienced the peer pressure to smoke a cigarette. It is often thought of as the “cool thing to do”. When people start smoking, they don’t think about the long-term consequences, financial or otherwise. What are the health risks? How much is involved financially, both in the short term and long term.

In 2012 the average cost of a pack of cigarettes in Illinois is $7 ($10 in Cook County). An estimated 20.2% of the people in Illinois smoke. Putting the facts together can lead a person to think twice about smoking, especially if they start at a young age, their teen years.

The following questions are designed to help you gain a better understanding of the various costs of smoking.

Please answer the following questions:

1. If an Illinois resident smoked 2 packs per day, how much would one year of cigarettes costs, assuming an average of $7 per pack? If the person stopped smoking after year 1, what would the future value of the one year of smoking cigarettes be 40 years from now, assuming a 3% interest rate, monthly compounding, and the cost of the cigarettes only?

   Hint, use the exponential growth function of $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. Now assume a person continues smoking 2 packs of cigarettes a day for 40 years. Calculate the lifetime total of the costs of these cigarettes, using the exponential growth model for compound interest. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ ***Special Note:*** For each additional year you will need to add the previous year’s total to the new year’s cost of cigarettes to determine the new P value. It is recommended you use a spreadsheet software (i.e. Excel) to get you started.

   P=initial principal (in this case start with annual costs)

   r = annual interest rate (in this case we will use 3%, assuming we can use the money we would have spent on the costs of smoking and invested it in a fund returning 3% annually, which is very low for long term investment)

   n = number of times per year the interest is compounded (use 12, for 12 months)

   t = the number of years we compound our money (when determining one year at a time, as is the case with #2, use “1” for your “t”).

3. Based only on the price of cigarettes, why is your answer in # 2 a large underestimate?
4. Besides the cost of buying cigarettes, what are some of the other costs? What is this cost not only to the smoker but also to the family, his/her job, and family at large?
5. I’m sure you will be amazed by the total costs involved. If a person didn’t smoke, what charity could they have done with it? How might it have been used to benefit society?

Up in Smoke

To the teacher:

The mathematical skills covered in this lesson include:

- Calculating compound interest;
- Using a spreadsheet to calculate the cost of smoking over many years.
- This project is designed for Algebra 2 students in their second semester. The first year calculation for #2 is calculated by assuming the full $5,110 is the initial principal. After that, I used the compounded interest formula and added the previous year's amount. Additional details are found in the answers below.

Answers:

1. Part One - $5,110. This is found by $7 x 2 x 365 = $5110.
    a. Part Two - $16,940.41. Use the information given and plug into the formula.
    b. $A = 5{,}110\left(1 + \left[\left(\frac{.03}{12}\right)\right]\right)^{12*40}$
2. After 40 years of smoking, the final cost is $400,784.48. See spreadsheet for details.
    a. Spreadsheet tips: FYI – most high school students have been taught how to perform these commands in a spreadsheet. If you need assistance with building your spreadsheet, you can do the following. When using Microsoft Excel, place the figures and formulas in the following cells:

Cell	Data	Notes / formula used
B1	$7	Avg cost of pack of cigarettes
B2	2	Packs per day
B3	365	# of days per year
B4	$5110	Use the formula "=B1*B2*B3"
G1	.03	Annual interest rate
G2	12	Number of times interest is compounded per year
B15	$5265.43	Use the exponential growth formula with the appropriate cells. =$B$4*(1+($G$1/$G$2))^($G$2*1)
B16	$10,691	Use the exponential growth formula with the appropriate cells, however add the amount from the cell above, cell B15 =(B15+$B$4)*(1+($G$1/$G$2))^($G$2*1)
B17 – B54		Copy the formula from B16 to B17 – B54. An easy way to do this is to click on cell B16. Move your mouse to the bottom right

corner of that cell until you get a little "+" sign. Click and drag down to B54. This will copy the formula, referencing all of the proper cells.

3. It is likely that the price of cigarettes will rise each year, probably faster than inflation. A pack of cigarettes in 4 years may cost $8 or more.
4. Besides the cost of the cigarettes, there is the added cost of healthcare. Smokers are more likely to suffer from various cardiovascular diseases – stokes, heart attacks, etc. When a parent or spouse dies or is chronically ill, there is a cost to the family. Smokers are probably more likely to miss work as the problems due to smoking catch up with them – resulting in lower productivity. The cost of smoking is not just to the individual but to the family at large – from the family to the health care system to taxpayers.
5. Answers will vary. i.e. they could have purchased a nice home, donated to "Help My Starving Children" and saved many lives.

Year 1	$5,265.43	Year 21	$151,671.16
Year 2	$10,691.00	Year 22	$161,549.81
Year 3	$16,281.61	Year 23	$171,728.93
Year 4	$22,042.25	Year 24	$182,217.66
Year 5	$27,978.11	Year 25	$193,025.41
Year 6	$34,094.52	Year 26	$204,161.88
Year 7	$40,396.96	Year 27	$215,637.09
Year 8	$46,891.10	Year 28	$227,461.32
Year 9	$53,582.77	Year 29	$239,645.20
Year 10	$60,477.96	Year 30	$252,199.67
Year 11	$67,582.88	Year 31	$265,135.99
Year 12	$74,903.91	Year 32	$278,465.78
Year 13	$82,447.61	Year 33	$292,201.01
Year 14	$90,220.75	Year 34	$306,354.00
Year 15	$98,230.33	Year 35	$320,937.48
Year 16	$106,483.53	Year 36	$335,964.53
Year 17	$114,987.75	Year 37	$351,448.63
Year 18	$123,750.64	Year 38	$367,403.71
Year 19	$132,780.06	Year 39	$383,844.07
Year 20	$142,084.11	Year 40	$400,784.48

Excel Spreadsheet can be obtained by contacting the St. Ignatius College Prep Math Department, or by emailing sandie.bulmann@ignatius.org.

Additional Smoking Statistics

Federal Statistics

(http://www.tobaccofreekids.org/research/factsheets/pdf/0072.pdf)

Tobacco Use in the USA

- High school students who are current (past month) smokers: 18.1% or 3.4 million [Boys: 19.9% Girls: 16.1%]
- High school males who currently use smokeless tobacco: 12.8% [Girls: 2.2%]
- Kids (under 18) who try smoking for the first time each day: 4,000
- Kids (under 18) who become new regular, daily smokers each day: 1,000+
- Kids (3-19) exposed to secondhand smoke: 50.2% or 32 million
- Workplaces that have smoke-free policies: 75.1%
- Packs of cigarettes consumed by kids each year: 800 million (roughly $2.0 billion per year in sales revenue)
- Adults in the USA who smoke: 19.3% or 45.3 million [Men: 21.5% Women: 17.3%]
- Deaths & Disease in the USA from Tobacco Use
- People who die each year from their own cigarette smoking: approx. 400,000
- Adult nonsmokers who die each year from exposure to secondhand smoke: approx. 50,000
- Kids under 18 alive today who will ultimately die from smoking (unless smoking rates decline): 6,000,000+
- People in the USA who currently suffer from smoking-caused illness: 8.6 million
- Smoking kills more people than alcohol, AIDS, car accidents, illegal drugs, murders, and suicides combined, with thousands more dying from smokeless tobacco use. Of all the kids who become new smokers each year, almost a third will ultimately die from it. In addition, smokers lose an average of 13 to 14 years of life because of their smoking.

The statistics are astounding, but what about the costs to society? Not only does a person in Illinois pay and average of $7 per pack of cigarettes ($10 in Cook County), but they and society also incur healthcare costs, productivity loss, and an increase in taxes to name the top costs involved. According to the research obtained from www.tobaccofreekids.org, the costs our society and each smoking individual can occur is extremely high.

Tobacco-Related Monetary Costs in the USA

- Total annual public and private health care expenditures caused by smoking: $96 billion
- Annual Federal and state government smoking-caused Medicaid payments: $30.9 billion
- [Federal share: $17.6 billion per year. States' share: $13.3 billion]
- Federal government smoking-caused Medicare expenditures each year: $27.4 billion
- Other federal government tobacco-caused health care costs (e.g. through VA health care): $9.6 billion
- Annual health care expenditures solely from secondhand smoke exposure: $4.98 billion
- Additional smoking-caused health costs caused by tobacco use include annual expenditures for health and developmental problems of infants and children caused by mothers smoking or being exposed to second-hand smoke during pregnancy or by kids being exposed to parents smoking after birth (at least $1.4 to $4.0 billion). Also not included above are costs from smokeless or spit tobacco use, adult secondhand smoke exposure, or pipe/cigar smoking.
- Productivity losses caused by smoking each year: $97 billion [Only includes costs from productive work lives shortened by smoking-caused death. Not included: costs from smoking caused disability during work lives, smoking-caused sick days, or smoking-caused productivity declines when on the job.]
- Annual expenditures through Social Security Survivors Insurance for the more than 300,000 kids who have lost at least one parent from a smoking-caused death: $2.6 billion Other non-healthcare costs from tobacco use include residential and commercial property losses from smoking-caused fires (about half a billion dollars per year) and tobacco-related cleaning & maintenance ($3 billion).
- Taxpayers yearly fed/state tax burden from smoking-caused gov't spending: $70.7 billion ($616 per household)
- Smoking-caused health costs and productivity losses per pack sold in USA (low estimate): $10.47 per pack
- Average retail price per pack in the USA (including sales tax): $5.29.

15. Using a Probability Simulation to Test Fairness

In 2012, female Chief Executive Officers of Fortune 500 Companies[4] reached the highest number historically: 18 female CEOs. Previously there have never been more than 16 female CEO's for Fortune 500 firms. While this is an increase, still only 3.6% of these companies are headed up by women. This gap goes beyond the position of CEO's. Only 15% of Board of Director seats were held by women in these companies and 12% of the firms had no women at all serving on their board[5]. This gap persists despite the fact that females are graduating from college in greater numbers than males. In the business world, this gap is called the glass ceiling – it is not visible, but some people are stopped from rising because of gender, race, or other characteristics.

Fortune 500 companies are not the only place a gender gap shows up. At my high school, I noticed that males tend to serve as the elected officers on the National Honors Society. For example, last year, all four of the NHS officers were boys, although 82 of the 153 NHS members were female. Does this suggest some type of bias towards boys in leadership, or might it just be due to chance? A probability simulation can be used to test the probability of 4 males being selected out of a group of 82 females and 71 males. If you have a TI-84 or similar calculator, you will use your random number generator to randomly pick 4 numbers from 1 through 153.

Here are the steps:

- Go to the MATH command and select PRB (probability)
- Under PRB, pick 5: randInt(
- Put in the following parameters: randInt(**1, 153, 4).** We use 153 to represent the 153 members. Numbers from 1 to 153 are selected randomly and 4 is the number of officers being selected.
- Hit enter and the calculator will generate 4 numbers between 1 and 153. Let the numbers 1-71 represent a boy and 72-153 represent a girl.
- Count the number of boys and girls for a trial. For example, if you get (92, 45, 79, 24), this represents two boys (45 and 24) and two girls. This is one trial. If any number comes up twice in a trial, ignore this trial and repeat it.
- Carry out 10 trials. Out of the ten trials, how often did you get numbers representing 4 boys?

The calculator's random generator functions make simulations easy. But you can also use a random number table or a computer's random generator function.

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[4] Fortune 500 companies are the 500 largest U.S. corporations.

[5] USA Today, June 2, 2012.

Answer the following questions:

1. Based on this simulation that you ran 10 times, what is the probability of randomly selecting a group of 4 that are all boys? (On what percent of the trials did you select 4 boys?) If possible, pool your results with the class to have a larger sample.

2. Do your results suggest whether there might be some bias towards electing boys as NHS officers? Why or why not?

3. The NHS officers are not selected by the teachers but by NHS members, and for the past few years, the majority of the officers have generally been boys -- even though more than half of the members are females. Can you think of some reasons why this might be?

4. In the instructions, you were told to ignore any trial where the same number came up twice, like (12 72 131 72). Why must you ignore it? What would this represent?

5. This type of simulation can be used to test bias in selection in various situations. For example, imagine a company with 60 female employees and 30 male employees. The boss selects 4 males and 1 female to serve on a committee looking at family leave policy. He claims there was no bias, that it was simply a random selection. Devise a simulation to check his claim that it was due to chance. Carry it out and comment on his claim.

Using a Probability Simulation to Test Fairness

To the teacher:

The mathematical skills covered in this lesson include:

- Using random numbers to carry out a simulation.
- Computing probability and using it to draw a conclusion.
- Planning a simulation that models a situation in real-life.
- Reasoning about mathematics in a real situation, applying the results, and communicating about conclusion.

Answers:

1. Answers will vary. When I did the simulation 20 times, I got 4 males once out of 20 trials: five percent of the trials. The actual probability is 4.427%. This is (71/153 * 70/152 * 69/151 * 68/150. While this will occur occasionally due to chance, it is not very likely.
2. Given that boys have been in the majority as NHS officers for several years, it suggests that something is going on that favors boys for these executive positions.
3. Bias might occur if boys are seen more as leaders, and so both boys and girls vote for them. But it may also be that girls are less likely to seek out these positions. This can be a self-imposed bias.
4. Since each number represents a particular student, you cannot pick him or her twice. Once a number representing a student is picked, it cannot be used again.
5. Answers will vary. Check for incorrect simulations. A simple one is the following:
   Use the random number generator to generate 5 numbers from 1 through 90. Let 1 -60 represent females and 61 – 90 represent males. Pick 6 random numbers, skipping repeats: RandInt(1,90, 6).

   I ran my simulation 20 times and only got 4 or more males 5% of the time. A committee of 4 or more males is not likely to occur by chance alone on one trial, suggesting there might have been bias in the selection process.

   Make sure that the students pick simulations that match the probabilities, that deal with repeats, and other considerations in correctly modeling a simulation.

16. Social Mobility (Probability and Matrix Multiplication)

Something about Nicole set her off from the others in that Algebra 1 class. Perhaps it was the leadership she showed. Or her curiosity about each new topic, eager to learn and do her best each day. And certainly she was doing well in her studies, in the top quartile of her class. If her teacher, Father Thul, had to pick the person from that group most likely to succeed, Nicole would have been at the top of his list.

When he found Nicole's name on his Geometry roster the following year, he was simply delighted. But it wasn't long before Fr. Thul noticed the change. 77, 60, 73, 64, 67, and 68 were her percentage test scores at the end of the first quarter. When he asked her why she was doing poorly and how he could help, she would only promise that she would bring her grades up. But the test and quiz grades continued low, and Nicole began to miss school more and more.

With the help of the office, her teacher checked up on Nicole and found that she lived in one of the poorest sections of the city, where gang violence was common and drugs plagued the streets. Nicole had four siblings, no father in the home, and her mother was unemployed with a history of drug abuse. Nicole sometimes had to stay home to take care of her brothers and sisters or help her mother. As she fell further behind, she began to give up on school. Perhaps Nicole had given up on succeeding – as a sophomore at the age of 14. Nicole dropped out of school by the spring of her sophomore year, and her teacher wondered about her chance of now escaping poverty.

Upward mobility is part of the American dream. Children hope to do better than their parents, and most parents help their children to climb the social ladder by providing educational opportunities – rising from lower class to middle class, or from middle class to the upper class. But for some people, poverty is difficult to escape.

Sociologists use different mathematical models for studying social mobility, one of which is a chain of matrices that involve probabilities. Consider the following matrix, which we will call Matrix A:

Next generation

Present generation

Class	Lower	Middle	Upper
Lower	.6	.3	.1
Middle	.1	.8	.1
Upper	.1	.2	.7

In this model, the probabilities of classes (lower, middle, and upper) are shown for the children of the present generation in some city. For example, the probability that a person in the lower class family will have a child that remains in the lower class is 60%, a child that moves to the middle class is 30%, and a child that reaches the upper class is 10%. For a middle-class family, the probability that they will have a child who falls into the lower class is 10 percent. Can you see these predictions in the matrix? Matrix A, with its probabilities, is called a transition matrix. The movement of future generations can be modeled by repeated matrix multiplication in a mathematical modeling process called Markov Chains.

Please answer the following questions:

1. Write the matrix that models the changes in generations in standard matrix form.
2. What is the number in the second row, third column? What does it represent?
3. Notice the sums of each row. What are these sums and why must this be true for the matrix to be valid? Why is this not true for the column sums?
4. Using your calculator, or by hand, find A X A or A^2. The resulting matrix gives the probabilities that a person will have a child that reaches the relevant class in two generations. For example the cell in row 1, column 2 shows the probability a person in the lower class will have a grandchild that reaches the middle class. (This probability will be 44% if you multiplied correctly.)
5. Based on the matrix, what is the probability that Nicole will escape the lower class? What is the probability that one of Nicole's children will reach the middle or upper class? Be sure to show or explain how you got your answer.
6. Now find A^4. What does the resulting matrix represent?
7. Let the row vector, B = [.4, .5, .1] represent the initial population distribution in the city being modeled. That is, 40% of the population is lower class, 50% is middle class, and 10% is lower class. Find B x A showing how you did the computation for the entry in row 1, column 1. What does the product matrix represent? Pick a couple of numbers from the matrix and explain what they tell us.
8. When you multiply the matrices together, what assumption are you making about the probabilities? How reasonable is this?
9. What are some causes of generational poverty, and what are some factors that help people escape poverty?

   Extension: Some Markov Chains display long-run equilibrium behavior. After many generations, the probabilities do not change. Do repeated multiplications of this transition matrix result in a transition matrix? If so, what is it and what do the numbers in the matrix represent?

Social Mobility

To the teacher:

The mathematical skills covered in this lesson include:

- Matrix multiplication and analyzing the results;
- Computing and interpreting probabilities;
- Using mathematical models to make long-term predictions and interpreting the reasonableness of these models.

Answers:

1. $$\begin{matrix} .6 & .3 & .1 \\ .1 & .8 & .1 \\ .1 & .2 & .7 \end{matrix}$$

2. The number is .1. The model says that the probability that the child of middle-class families reach the upper class is 10%.

3. The rows must add up to 1 or 100% because a child of a given class must reach lower class, middle class, or upper class. There are no other possibilities. For the rows, these tell us what classes children end up in. There is not reason for lower-class (or any column) to add up to 100%. Under the right conditions, the majority of children could move into the middle and upper classes.

4. $$A \text{ X } A = \begin{matrix} .40 & .44 & .16 \\ .15 & .69 & .16 \\ .15 & .33 & .52 \end{matrix}$$

5. The probability that Nicole will escape the lower class – reaches the middle or upper class – is .3 + .1 = 40%. This is the sum of the second and third column in row 1. The probability that one of her children will escape the lower class if she remains in the lower class is
.44 + .16 = 60%. This is the complement of the entry in the first row and first column in the matrix A X A.

6. This represents the probabilities for the 4^{th} generation, Nicole's great-grandchildren.

7. [B]*[A] = [.3, .54, .16]. The first cell is computed as .4*.6 + .5 * .1 + .1 * .1 or .30. This tells us that in the next generation, 30% of the population for this city will be lower class.

8. You are making the assumption that the probability does not change over time. It is not reasonable that there will be no change in probabilities, but if the changes are small, the answers still may be reasonable estimates.

9. Answers will vary. One of the most important factors of course is a good education, which is supported by a family that values school and success. Age of mother, number of books in the house, and other variables are also predictors of success. Children in poverty certainly find it more difficult to reach the middle class or higher than do middle class children.

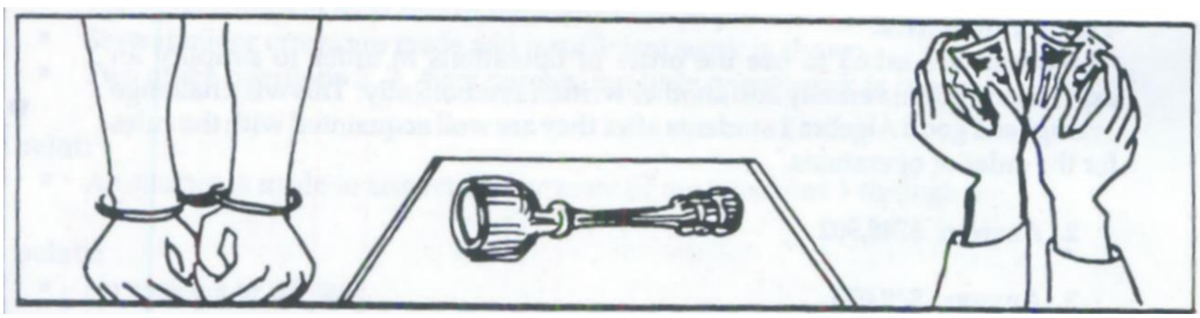
Optional question: If you multiply the transition matrix enough times, the resulting equilibrium matrix is:

.2	.55	.25
.2	.55	.25
.2	.55	.25

This suggests that in the long run, the probability of a distant descendent being in the lower class is the same (20%) no matter what class you are in. 20% of all future descendents will be lower class. Of course, this is based upon a stable model.

17. Decisions and Justice in the Court System:
A Statistical Perspective

In 1991, a 14-year old girl was sexually assaulted and murdered after leaving her grandmother's house in Illinois. A teenager brought in for questioning confessed, and he implicated four others teens including Jonathan Barr who was 15 at the time. Jonathan was convicted and sentenced to 85 years in prison. He was imprisoned for 15 years before he and the others were exonerated of the crime. DNA evidence showed the sample from the crime scene matched that of a violent serial offender. Five innocent boys had been convicted and imprisoned while the murderer remained free to commit other crimes. Finally the boys were freed with the help of The Innocence Project[6], a legal aid group that helps those imprisoned incorrectly, especially by using DNA evidence that had been unavailable or ignored.



The trial and jury system in the United States is a hallmark of the rights and freedoms of its citizens. All citizens accused of a crime are entitled to an attorney and a fair trial before their peers. Judges preside over the trial, appeals can be brought to higher courts, evidence must show the plaintiff's guilty beyond a reasonable doubt, and all jurors must agree on the plaintiff's guilt in order for him or her to be convicted. Despite all of these safeguards, we know that people who are guilty of a crime are sometimes found *not guilty*. And innocent people, like Jonathan Barr, are sometimes found guilty, spending years in jail before they are released. We might wish to avoid all errors, but can we?

Jury trials resemble statistical tests. In statistics, we have two hypothesis: the *null hypothesis* and the *alternative hypothesis*. We accept the null hypothesis unless there is

[6] See the Innocence Project at www.uis.edu/innocenceproject/ for more information on this case and other cases on which they work, helping to free the wrongly convicted.

sufficient evident to reject it in favor of the alternative hypothesis. For example, in testing a new cancer drug, the hypotheses might be:

Null hypothesis: The new drug is not better than existing drugs;

Alternative hypothesis: The new drug is better at fighting cancer.

If, after clinical trials, there is evidence that the new drug is more effective, researchers reject the null hypothesis and accept the alternative, saying the new drug is more effective. If there is not enough evidence that it is an improvement, researchers will not reject the null hypothesis. The new drug is not an improvement over previous ones.

There are 2 types of errors that can be made. In a Type 1 error, we reject the null hypothesis and say the drug works better, when in fact it really is not more effective. In a Type 2 error, we say the drug is not better, when in fact it is an improvement, and we should market it. Although we might like to eliminate both types of errors, generally if we reduce the probably of a Type 1 error, we can increase the probability of a Type 2 error. And vice versa.

In the U.S. jury system, the null hypothesis is that the plaintiff is **not guilty**. Jurors must find sufficient evidence to reject the null hypothesis and say he is guilty. While we might hope no guilty people go free and especially that no innocent people end up in jail, Type 1 and Type 2 errors occur, and perhaps can never be avoided completely. This is illustrated in the table below.

		The Court Decision	
		Not Guilty	Guilty
The Truth	Not Guilty	Correct decision	Type 1 error
	Guilty	Type 2 error	Correct decision

In reading the table, note that if the person is found **not guilty**, when the truth is that he/she is **guilty**, this is a Type 2 error. The table contains two correct decisions and two types of errors. The same is true in statistical decisions.

Please answer the following questions:

1. In our jury system, what are the null and the alternative hypotheses?
2. A Type 1 error would be finding an innocent person guilty of a crime he or she did not commit? What is a Type 2 error?
3. What type of error do you think would be worse? Explain your reasoning.
4. A belief of our system is that a person is innocent until proven guilty. How does this correspond to the null and alternative hypotheses in the court?
5. In some types of crimes, especially those involving violence, innocent people have been imprisoned, only to be freed latter when DNA and other evidence exonerated them. This is especially true when the defendants come from poverty and have little education. Why do you think this happens?
6. Given that innocent people are jailed, why not make it harder to find someone guilty? Discuss this in terms of Type 1 and 2 errors.

Decision and Justice in the Court System

To the teacher:

The mathematical skills in this project include:

- Statistical concepts, specifically hypotheses testing in AP Statistics courses.
- Understanding types of errors in statistics and in the context of trials.
- Interpreting the results of errors in the justice system.

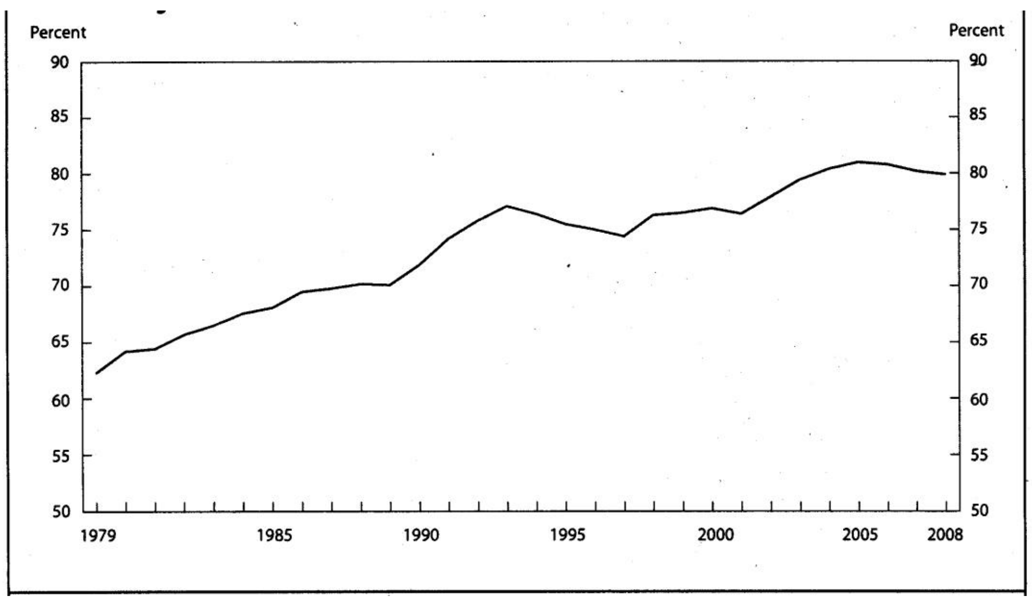
Answers:

1. As stated in the reading, the null hypothesis is “the person is not guilty.” The alternative is “the person is guilty.” From a statistical point of view, if the person is not guilty, we would not say they are innocent. There is just not sufficient evidence to prove guilt. [Note: this is not the case in all countries. In some countries, a person might be considered guilty and must prove their innocence.]
2. A Type 2 error would be finding a person not guilty when in fact he or she is. This is when we let a guilty person go free.
3. Students answers may vary. Jailing an innocent person ruins his life, the life of his family, and leaves the guilty person free to commit crimes (type 1 error). Releasing a guilty person fails to punish a criminal and to protect the victim and society (type 2 error). But given our belief that a person is innocent unless found guilty **beyond a reasonable** doubt, it would seem a Type 1 error is worse. It is said of our court system that we would rather see 99 guilty men go free than 1 innocent man jailed.
4. The null hypothesis is that a person is not guilty. There must be strong evidence against this to reject the null hypothesis and confirm guilt. Given that most juries have 12 members and all must agree, it would seem that a level of rejection must be less than 1/12 or .083. This corresponds somewhat to statistical tests where a p-value of less than .05 is typical for establishing statistical significance.
5. Students’ answers will vary. Poor people and those with less education may have less access to good representation. Public defenders with an overload of cases may not be able to represent the defendants as well. Also, in cases involving extreme violence, crimes against children, sexual assault, and other heinous offenses, police and prosecutors may work harder to put someone away. In these types of serious cases, jurors may be more sympathetic to the victims, the police, and the prosecutors.
6. By making it harder to convict someone, we will most likely have to release more persons who are guilty. If we make it harder to make a type 1 error, we are more likely to make a type 2 error.

18. Equal Work Does Not Mean Equal Pay: Differences in Men's and Women's Pay

We don't expect all jobs to pay the same. On average, high school graduates will earn more than dropouts. College graduates will earn more than high school graduates -- both annually and over the course of their life. But some differences in pay bring up issues of bias and fairness. For example, women in the United States earn less than men. Figure 1 shows women's median weekly earnings as a percent of men's for full-time workers from 1979 to 2008.[7] While the gap has narrowed in this time period, from earning only 62% of men's salaries in 1979 to 80% in 2008, women still lag behind men in earning.

Figure 1: Women's earning as a percent of men's, full-time wage and salary workers, 1979 – 2008 annual averages

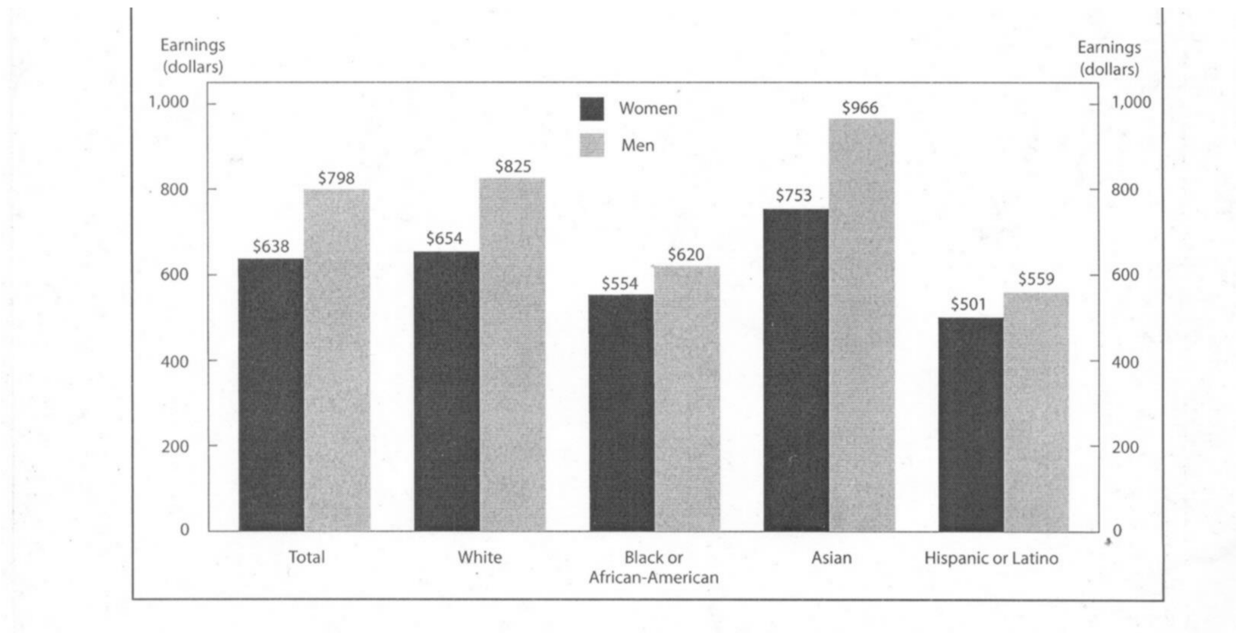


[Note: While the arithmetic mean is often used as an average, median salaries are used by the U.S. Bureau of Statistics as in these comparisons.]

[7] From "Highlights of Women's Earnings in 2008," The US Bureau of Labor Statistics.

The statistics here include only full-time workers. Of the full-time work force, 44% were women. In comparison, more than twice as many part-time workers are females. Part-time salaries, generally much lower, are not included in the charts shown. Male-female differences by ethnicity are shown in Figure 2.

Figure 2: Median weekly earnings of full-time and salary workers by sex, race, and ethnicity, 2008 annual averages



One fact that does not show up in these statistics is 13.1% of households are headed up by a single women (U.S. Census Bureau, 2010), sometimes with little or no support from the father. Almost half of these families live below the poverty line, and nearly 1/5 had no health insurance. These lower earnings, when a woman is the head of the household, can adversely affect the children who may live in poverty and lack good health care.

Please answer the following questions:

1. It is stated that women currently earn about 88% of men's salaries. Confirm this using the information from the graph.
2. How much does the average woman working full-time earn in a year? (Assume 52 weeks of pay.) How much more does the average man earn in a year?
3. What do you think accounts for the difference in men-women's salaries, both in 1980 and over time? Come up with 3 reasons and comment on them. Why has the gap narrowed?

4. Chart 2 shows salary differences by race/ethnicity. What patterns do you see? What percent is the weekly earning of Hispanic females compared to White males?

5. The median rather than the arithmetic mean is generally used in comparing salaries, housing prices, and similar statistics. Why do you think that the median is used? Would you expect a larger or smaller difference if the arithmetic mean was used for salaries? Why?
6. In 2007-2008, the U.S. went into a recession with over 10% of the work force unemployed. How do you think that Figure 1 would appear for men and women if the graph was extended beyond 2008? Why?
7. Based on the graph shown here, if the trend continues, when might the gap between men and women's pay disappear? Explain how you got your answer.

Equal Work Does Not Mean Equal Pay

To the teacher:

The main mathematical skills covered in this lesson include:

- Reading and using information from time plots and bar graphs;
- Using ratios and percents in computations and comparisons;
- Reasoning and communicating about mathematical concepts and social justice issues.

Answers:

1. 638/798 = .7995

2. 638 * 52 = 33,176. Men's average salary is $ 41,470. The difference is around $8294.

3. Students may come up with a number of reasons:

   - Traditionally women have worked at lower paying jobs compared to men.
   - Even professional women have tended toward service jobs like teaching and nursing that have paid less than other professions in the past.
   - The "glass ceiling" has kept women from rising to higher positions in corporations.
   - When women have children, they might leave the work force for some time. If a salary is based on years worked, leaving the work force to stay home can have an effect.

   However, a recent study shows that even when women's choices (type of job, work fewer hours) are taken into account, 1/3 of the difference is still unexplained.

4. In all groups, women make less than men. The biggest gap is with Asians, where men make $213 more per week than women. For minorities, like African Americans and Hispanics, the salary gap is the lowest. African Americans and Asian men make less than the average female salary overall. Hispanic females are the lowest paid. On average they earn $ 297 less than a White male per week. This is 62.7%.

5. Means are more likely to be changed by outliers in either direction. For example, a few people may have annual earnings of millions of dollars. This will pull the mean toward higher numbers, but will not affect the median. The median tells us the middle number, which is a better average to use when there are some extreme values (outliers.)

6. Student's answers will vary. All workers suffered in the downturn and recession, but it is generally the lowest paid that suffer the most, so the gap may have increased. With

decreases in salaries for many workers, women, with lower salaries, may have seen larger percentage increases in earnings. However, it may also be that more males were in jobs (middle-management) that had cuts due to the downturn.

7. Answers will vary depending on students' approaches. If the linear trend continues, it looks like gap will disappear around 2040 or so. I drew a trend line that seemed to reach 100% (same salary for men and women) around 2040. However the main point of the question is that one should be cautious about extrapolating a trend so far into the future.