Quiz #1

Calculus I – Math 161.001 – Spring 2012

Name: _____

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1. ~~(2~~ pts) Suppose f is an even function and g is an odd function.

   - (T / F) f/g is odd. $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$
   - (T / F) $g \circ g$ is odd. $g(g(-x)) = g(-g(x)) = -g(g(x))$
   - (T / F) $(f+2)g$ is odd. $[(f+2)g](-x) = (f+2)(-x) \cdot g(-x) = (f(-x)+2)g(-x) = (f(x)+2)(-g(x)) = -[(f+2)g](x)$

2. (3 pts) The half-life of a radioactive substance is 12 hours. There are 6 grams present initially.

   - Express the amount of substance remaining as a function of time t.
   - When will there be 1 gram remaining?

- $P(t) = P_0\left(\frac{1}{2}\right)^{t/12}$ $\quad P_0 = 6.$

- $1 = 6(1/2)^{T/12}$ solve for T.

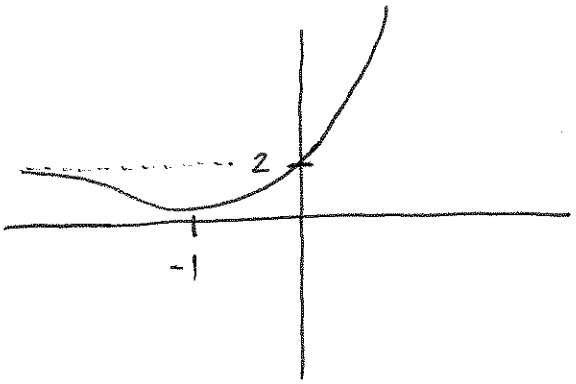
$$\ln(1/6) = \ln\left((1/2)^{T/12}\right) = \frac{T}{12} \cdot \ln\left(\frac{1}{2}\right)$$

$$\frac{\ln(1/6)}{\ln(1/2)} = \frac{T}{12}$$

$$\frac{0 - \ln(6)}{0 - \ln(2)} = T/12 \qquad T = 12 \cdot \frac{\ln(6)}{\ln(2)}$$

3. (2 pts) Explain why the curve $y = xe^x + 2$ does not represent a one-to-one function f on $\mathbb{R}$, then give a restricted domain on which it is one-to-one. Finally, sketch a graph of f^{-1}.

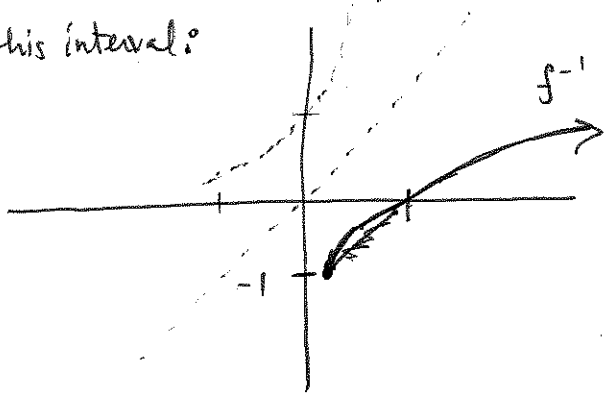
Here is a plot of f:



Doesn't pass the horizontal line test (for, say, the line $y = 1.9$)

From $x = -1$ to ∞, f is one-to-one

f^{-1} is the reflection across $y = x$ on this interval:



4. (3 pts) Sketch a picture of a function f with domain $(1, 2)$ and range $(-\infty, 3) \cup [5, \infty)$. Then, sketch a picture of $2f(x-1) - 1$.