Quiz #2

Calculus I – Math 161.001 – Spring 2012

Name: ____________________________________

1. (2 pts) Convince me, using an epsilon-delta argument, that $\lim_{x\to 3}(2x+1) \neq 6$.

$|(2x+1)-6| < \varepsilon$

$-\varepsilon < 2x-5 < \varepsilon$

$\frac{5-\varepsilon}{2} < x < \frac{5+\varepsilon}{2}$

For ε really small, these are numbers centered around $5/2 = 2.5$, say $(2.4, 2.6)$, which cannot happen if we demand that $0<|x-3|<\delta$. For this would be $x \in (3-\delta, 3+\delta)$. In particular, #s ~~x~~ larger than 3 should also satisfy $f(x)$ is ε-close to 6. Impossible.

2. (4 pts) If $\lim_{x\to 1} f(x) = 3$ and $\lim_{x\to 3} g(x) = 1$, then compute $\lim_{x\to 1}\Big(10 + g(f(x)) - f(x)\Big)^{1/3}$.

(Assume f is continuous @ $x=1$ and g is continuous at $x=3$.)

$$\lim_{x\to 1}\left(10 + g(f(x)) - f(x)\right)^{1/3} = \left(\lim_{x\to 1} 10 + \lim_{x\to 1} g(f(x)) - \lim_{x\to 1} f(x)\right)^{1/3}$$

$$= \left(10 + g\left(\lim_{x\to 1} f(x)\right) - 3\right)^{1/3}$$

$$= \left(10 + g(3) - 3\right)^{1/3} = \left(10+1-3\right)^{1/3}$$

$$= 8^{1/3} = \underline{\underline{2}}$$

3. (4pts) Find values of a and b so that the function given by

$$f(x) = \begin{cases} ax^2, & x \leq -2 \\ bx - a, & -2 < x \leq 0 \\ (\tan x)/x & 0 < x < \pi/2 \end{cases}$$

is continuous at every point in its domain. (Use theorems and observations from class to evaluate any limits involved, not epsilon-delta arguments.)

Need:

① $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) = f(-2)$

② $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

① $a(-2)^2 = b(-2) - a$

② $b(0) - a = \lim_{x \to 0^+} \frac{\tan x}{x} = \lim_{x \to 0^+} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \left(\lim_{x \to 0^+} \frac{\sin x}{x}\right)\left(\lim_{x \to 0^+} \frac{1}{\cos x}\right)$

$= 1 \cdot 1 = 1$

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② $\Rightarrow$ $a = \underline{\underline{-1}}$

① + ② $\Rightarrow$ $-4 = -2b + 1$, OR $b = -5/-2 = \underline{\underline{5/2}}$

For all other points within $(-\infty, \pi/2)$, $f(x)$ is continuous by theorems from class. (polys. + ~~ratio~~ quotients of cont. fcns. are cont.)