Quiz #3

Calculus I – Math 161.001 – Spring 2012

Name: _____

1. (6 pts) Take the indicated derivatives of these functions using differentiation rules:

   - $\frac{d}{dt}(t^2 - 2t^{-1})$ $= 2t + 2t^{-2}$

   - $\frac{d^2}{dt^2}\left[(1+t)e^t\right] = \frac{d}{dt}\left[\,[1]e^t + (1+t)[e^t]\,\right]$

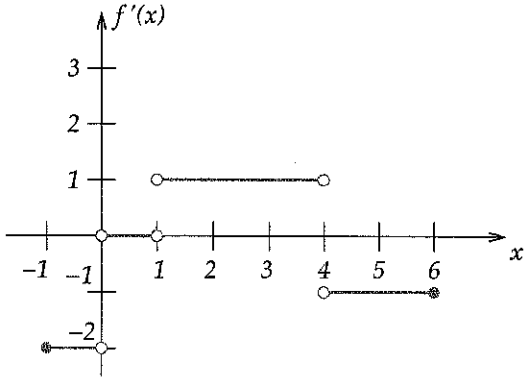
     $= e^t + [1]e^t + (1+t)[e^t]$

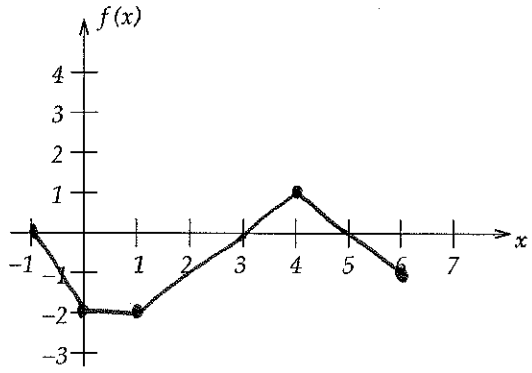
     $= e^t(3+t)$

   - $f'(x)$ if $f(x) = \cos x$

     $f'(x) = -\sin x$

2. (2 pts) The graph of a function f is made up of line segments, **connected end-to-end.** Sketch the graph of f if $f(1) = -2$, and $f'(x)$ is the graph given below at left.





3. (2 pts) *Tangents & Derivatives* Let $f(x) = \sqrt{2-x}$.

   (a) Using the limit definition of derivative, determine $f'(x)$.

   (b) Find the tangent line passing through the point $(1, 1)$ on the graph of $f(x)$.

(a) $\lim_{z \to x} \dfrac{\sqrt{2-z} - \sqrt{2-x}}{z-x}$

$= \lim_{z \to x} \dfrac{(2-z)-(2-x)}{(z-x)\left(\sqrt{2-z} + \sqrt{2-x}\right)}$

$= \lim_{z \to x} \dfrac{-(z-x)}{(z-x)\left(\sqrt{2-z} + \sqrt{2-x}\right)}$

$= \lim_{z \to x} \dfrac{-1}{\sqrt{2-z} + \sqrt{2-x}}$

$= \dfrac{-1}{2\sqrt{2-x}}$

Alternatively ...

$\lim_{h \to 0} \dfrac{\sqrt{2-(x+h)} - \sqrt{2-x}}{h}$

$= \lim_{h \to 0} \dfrac{\{2-(x+h)\} - \{2-x\}}{h\left(\sqrt{2-(x+h)} + \sqrt{2-x}\right)}$

$= \lim_{h \to 0} \dfrac{-h}{h\left(\sqrt{2-x-h} + \sqrt{2-x}\right)}$

$= \lim_{h \to 0} \dfrac{-1}{\sqrt{2-x-h} + \sqrt{2-x}}$

$= \dfrac{-1}{2\sqrt{2-x}}$

(b) $f'(1) = \dfrac{-1}{2\sqrt{2-1}} = \dfrac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \left(-\tfrac{1}{2}\right)(x-1)$

$y = -\tfrac{1}{2}x + \tfrac{3}{2}$