ABSTRACT

Degree of approximation of functions of bounded variation by some singular integrals

by

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The Picard, Poisson-Cauchy and Gauss-Weierstrass singular integers ([6],[1], p.403) are respectively given by

$$P(x,\xi) = (2\xi)^{-1}\int_{-\infty}^{\infty} f(x+t)\exp\{-|t|/\xi\}dt,$$
$$Q(x,t) = \frac{\xi}{\pi}\int_{-\pi}^{\pi}\frac{f(x+t)}{t^2+\xi^2}\ dt,$$
$$W(x,\xi) = \frac{1}{\sqrt{\pi\xi}}\int_{-\pi}^{\pi} f(x+t)\exp(-t^2/\xi)\ dt.$$

Khan[5] and Decba et al. [2] have studied error bounds for approximation of functions of class L_p by using Picard, Poisson-Cauchy and Gauss-Weierstrass singular integrals. Mohapatra and Rodriguez [6] have obtained results on the degree of convergence of above singular integrals for Hölder continuous functions. Further works have been carried out on the degree of approximation of continuous functions using singular integrals by Gal ([3],[4]).

Throughout the present work we assume that $f \in L(A)$, where $A = (-\infty,\infty)$ for Picard's singular integral and $A = (-\pi,\pi)$for the remaining two singular integrals.

We write

$$\phi_x(t) = \begin{cases} f(x+t)+f(x-t)-f(x+0)-f(x-s), & t \neq 0 \\ 0, \quad t=0 \end{cases}$$

$V_0^t(\varphi_x) =$ Total variation of ϕ_x over $[0,t]$.

In the present work, we obtain the following results on the rate of convergence of above singular integrals for functions of bounded variation.

Theorem 1: *If $f \in BV(-\infty, \infty)$ then for positive Δ, however large*

$$\left|P(x,\xi) - \frac{1}{2}\left\{f(x+0) + f(x-0)\right\}\right| \leq \frac{\Delta\xi^2}{\pi^2}\sum_{k=1}^{Z}(k+1)V_0^{\pi/k}(\phi_x) + O(\xi^{\Delta})$$

as $\xi \to 0+$, *where* $Z = [\pi/\xi]$.

Theorem 2: *If $f \in BV(-\pi, \pi)$ then*

$$\left|Q(x,\xi) - \frac{1}{2}\left\{f(x+0) + f(x-0)\right\}\right| \leq \left(\frac{1}{4} + \frac{1}{\pi}\right)\frac{\xi}{\pi}\sum_{k=1}^{Z}V_0^{\pi/k}(\phi_x) + O(\xi)$$

as $\xi \to 0+$, *where* $Z = [\pi/\xi]$.

Theorem 3: *(i) If f is of bounded variation in the neighborhood of at point x in $(-\pi, \pi)$, then for positive Δ, however large*

$$\left|W(x,\xi) - \frac{1}{2}\left\{f(x+0) + f(x-0)\right\}\right| \leq \left(\frac{2+\sqrt{2}}{\pi^2\sqrt{\pi}}\right)^{\xi}\sum_{k=Y}^{Z}(k+1)V_0^{\pi/k}(\varphi_x) + O(\xi^{\Delta})$$

as $\xi \to 0+$, *where* $X = [\pi/\sqrt{5}]$ *and* $Y = [\pi/\xi^{\beta}], 0 < \beta < \frac{1}{2}$.

(ii) If $f \in BV(-\pi, \pi)$ then the estimate given in case (i) holds for all $x \in (-\pi, \pi)$.

References

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