FORWARD COMPACTNESS

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ABSTRACT. A real function f is continuous if and only if $(f(x_n))$ is a convergent sequence whenever (x_n) is convergent and a subset E of $\mathbf{R}$ is compact if any sequence $\mathbf{x} = (x_n)$ of points in E has a convergent subsequence whose limit is in E where $\mathbf{R}$ is the set of real numbers. These well known results suggest us to introduce a concept of forward continuity in the sense that a function f is forward continuous if $\lim_{n\to\infty} \Delta f(x_n) = 0$ whenever $\lim_{n\to\infty} \Delta x_n = 0$ and a concept of forward compactness in the sense that a subset E of $\mathbf{R}$ is forward compact if any sequence $\mathbf{x} = (x_n)$ of points in E has a subsequence $\mathbf{z} = (z_k) = (x_{n_k})$ of the sequence $\mathbf{x}$ such that $\lim_{k\to\infty} \Delta z_k = 0$ where $\Delta y_k = y_{k+1} - y_k$. We prove that any forward continuous function defined on a forward compact subset of $\mathbf{R}$ is uniformly continuous, uniform limit of forward continuous functions is forward continuous, any forward continuous function is continuous. Some other results related to forward continuity and continuity are also obtained and some open problems are discussed.

REFERENCES

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