Loyola University Chicago
Math 475, Fall 2014

HW 11

Book Problems. (MacCluer)

Problem 1. Let $\|\cdot\|_s : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$\|(x_1, x_2)\|_s = \begin{cases} \|(x_1, x_2)\|_2 & \text{if} \quad x_1 x_2 \geq 0 \\ \|(x_1, x_2)\|_1 & \text{if} \quad x_1 x_2 \leq 0 \end{cases}.$$

Is this a norm? Find $m, M > 0$ so that, for every $x \in \mathbb{R}^2$,

$$m\|x\|_\infty \leq \|x\|_s \leq M\|x\|_\infty.$$

Problem 2. On c_0, let $\|\cdot\|_b$ be defined by

$$\|x\|_b = \sum_{i=1}^{\infty} \frac{1}{2^i}|x_i|.$$

Is $\|\cdot\|_b$ a norm? Is it equivalent to the supremum norm?

Problem 3. Consider $K = \left\{ f \in L_1[0,1] \,:\, 0 \leq f \leq 2, \int_0^1 f\,dt = 1 \right\}$ and $T : K \to L_1[0,1]$ given by

$$Tf(t) = \begin{cases} \min\{2f(2t), 2\} & \text{if} \quad 0 \leq t \leq 1/2 \\ \max\{0, 2f(2t-1) - 2\} & \text{if} \quad 1/2 < t \leq 1 \end{cases}$$

Show that $T(K) = K$, that T is an isometry from K to K, and that T does not have a fixed point in K (no $f \in K$ such that $Tf = f$.)