Book Problems. (MacCluer) 4.14, 4.17, 4.19, 4.22, 4.24

Problem 1.   Is there a bounded linear operator T on a Hilbert space l^2 so that every rational number between 7 and 11 is an eigenvalue of T? If yes, give an example. If no, prove it.

Problem 1'.   Is there a bounded linear operator T on a Hilbert space $L^2[0, 1]$ so that every rational number between 7 and 11 is an eigenvalue of T? If yes, give an example. If no, prove it.

Problem 2.   Find a nonzero compact and a nonzero non-compact bounded linear operator $T : l_2 \rightarrow l_2$ so that $T^3 = 0$.

Problem 3.   Let T be a linear operator on a Hilbert space H such that, for every $x, y \in H$,

$$\langle Tx, y \rangle = \langle x, Ty \rangle.$$

Show that T is a bounded operator.