Loyola University Chicago
Math 475, Fall 2014

HW 4

Reading: MacCluer, Chapter 2 and Appendix A; Bachman/Narici, Chapter 9 and 10.

Book Problems. (MacCluer) 1.21 a, b; 1.22

Problem 1. Let M be a subspace of a Hilbert space X. Prove that M is dense in X if and only if $M^{\perp} = \{0\}$.

Problem 2. Let c be the space of sequences of real numbers that converge. That is, $x \in c$ means that $x = (x_1, x_2, \dots)$ and $\lim_{j\to\infty} x_j$ exists. It is easy to verify that c is a vector space. For $x \in c$, define $\|x\| = \sup_{j\in\mathbb{N}} |x_j|$. Verify that this is a norm and that c with this norm is a Banach space.

Problem 3. Construct a function $f : [0, 1] \to \mathbb{R}$ which is discontinuous at every rational point in $[0, 1]$ and continuous at every irrational point in $[0, 1]$.