**Loyola University Chicago**
**Math 475, Fall 2014**

# HW 7

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**Book Problems.** (MacCluer) 3.11; 3.12; 3.14; 3.21;

**Problem 1.** Recall: in $l_1$, $x_n$ converges weakly to $x$ if, for every $y \in l^\infty$, $\lim_{n\to\infty} \langle x_n, y\rangle = \langle x, y\rangle$, where $\langle x, y\rangle = \sum_{n=1}^{\infty} x_n y_n$. Prove that, in $l^1$, $x_n$ converges weakly to $x$ if and only if $\|x_n - x\| \to 0$.

**Problem 2.** Let $X = Y = C[0,1]$. A mapping $T : X \to Y$ is called *causal* if, for every $\tau \in [0,1]$ and every $f, g \in X$, if $f(t) = g(t)$ for every $t \in [0,\tau]$, then $Tf(t) = Tg(t)$ for every $t \in [0,\tau]$.

- Show that $T$ given by $Tf(t) = (f(t))^2$ is causal.
- Show that $T$ given by $Tf(t) = f(1-t)$ is not causal.
- Find a couple of nontrivial and significantly different from one another examples of causal mappings from $X$ to $Y$, and a couple of nontrivial and significantly different from one another examples of not causal mappings from $X$ to $Y$. *(An example of a causal mapping like $Tf(t) = (f(t))^3$ is neither nontrivial nor sufficiently different from what you see above.)*

**Problem 3.** Let $X$ be the normed vector space $C^1[0,1]$ of continuously differentiable functions on $[0,1]$, with the sup norm $\|f\| = \max_{t\in[0,1]} |f(t)|$. Find a sequence of bounded linear functionals $T_n : X \to \mathbb{R}$ so that, for every $f \in X$, $\sup_{n\in\mathbb{N}} |T_n f| < \infty$ but $\sup_{n\in\mathbb{N}} \|T_n\| = \infty$. *(This will illustrate that the PUB fails if $X$ is not complete.)*