Loyola University Chicago
Math 475, Fall 2014

HW 9

Book Problems. (MacCluer) 4.5 and Read 4.1, 4.2

Problem 1. Let X be a Banach space with norm $\|\cdot\|$. Prove that, for any sequence $\{x_n\}$ in X, if $\sum_{n=1}^{\infty} \|x_n\| < \infty$ then $\lim_{k\to\infty} \sum_{n=1}^{k} x_n$ exists.

Problem 2. Let X be the Banach space $C[0,1]$ with the supremum norm. Let

$$M = \{f \in X \mid f(0) = 0\}.$$

Show that M is closed. Find an explicit formula for the quotient norm $\|[f]\|$ for $[f] \in X/M$. Find an isometric isomorphism from $\mathbb{R}$ to X/M.

Problem 3. Let X be the vector space $C[0,1]$ with the norm $\|f\|_1 = \int_0^1 |f(t)|\, dt$. Let

$$M = \{f \in X \mid f(0) = 0\}.$$

Show that M is not closed. Show that the "quotient norm" $\inf\{\|f - m\|_1 \mid m \in M\}$ is not a norm on X/M.

Problem 4. Prove Hölder's inequality:

$$\int_0^1 |f(t)g(t)|\, dt \leq \|f\|_p \|g\|_q,$$

where $f \in L^p[0,1]$, $g \in L^q[0,1]$, and $1/p + 1/q = 1$. *(It may help to recall how we did this in l^p.)*