Loyola University Chicago
Math 161, Spring 2010

Final Exam

Name (print): _____ **Signature:** _____

Please do not start working until instructed to do so.

You have 2 hours.

No calculators, iPhone's, laptops, or any other devices that do more than show time.

You must show your work to receive full credit.

You may use one double-sided 8.5 by 11 sheet of handwritten (by you) notes.

Problem 1. _____

Problem 2. _____

Problem 3. _____

Problem 4. _____

Problem 5. _____

Problem 6. _____

Problem 7. _____

Problem 8. _____

Total. _____

Problem 1. *(32 points total)* Find the following limits, derivatives, and integrals:

a. *(4 points)* $\lim_{x \to -\infty} \dfrac{x^3 - 5x^7 + 8x}{x^4 + 9 + 15x^7}$

b. *(4 points)* $\dfrac{d}{dx}\left(\cos(5x^2) + 3^x \ln x\right)^6$.

c. *(4 points)* $\lim_{x \to \infty} \dfrac{1 + \sqrt{x-3}}{x}$.

d. *(4 points)* $\lim_{t \to 0^-} 1 + \dfrac{2}{\sqrt[3]{t}}$

e. *(4 points)* $\dfrac{d}{dx}\displaystyle\int_{e^x}^{\tan^{-1}(x)} (x+1)^{777}\,dt$

f. *(4 points)* $\displaystyle\int \left(\frac{5}{\sqrt{1-x^2}} - x^7 + 11\right) dx$

g. *(4 points)* $\displaystyle\int_0^6 \left(x + \sqrt{36-x^2}\right) dx.$

h. *(4 points)* $\displaystyle\int \frac{x}{\sqrt{x+7}}\,dx$

Problem 2. *(8 points total)* Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x^2 + 1}$.

Problem 3. *(8 points total)* Find all horizontal and all vertical asymptotes of the function

$$f(x) = \frac{2 - e^x}{2 + e^x}$$

Problem 4. *(14 points total)* Let $g(x) = \frac{1}{4}x^4 - x^2$.

a. *(6 pts)* Find the critical points for this function. For each critical point, determine whether it is a local minimum, local maximum, or neither.

b. *(4 pts)* Identify the intervals on which g is concave up and concave down.

c. *(4 pts)* Find the absolute minimum and the absolute maximum of $g(x)$ on the interval $[-1, 3]$.

Problem 5. *(8 points)* Consider the function

$$f(x) = \ln x - x.$$

a. *(4 pts)* Write the left-endpoint Riemann sum for this function on the interval $[1, 3]$ with $n = 4$ subintervals. *(You do not need to evaluate the sum.)*

b. *(4 pts)* Circle the correct statement and provide a brief explanation of your answer.

- The left-endpoint Riemann sum is an underestimate of $\int_1^3 \ln x - x \, dx$.
- The left-endpoint Riemann sum is an overestimate of $\int_1^3 \ln x - x \, dx$.
- Neither of the two statements above is true.

Problem 6. *(10 points)* A rocket car is moving along a straight line. Laws of physics determine that the acceleration of the cart is given by the formula

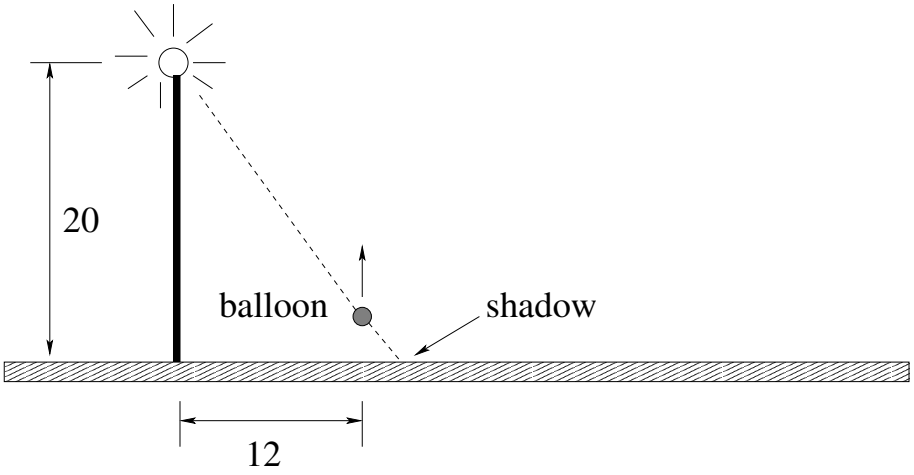
$$a(t) = 2t - 5$$

for $t \geq 0$. The velocity of the cart was measured at $t = 0$ to be $v(t) = 6$ meters/second.

a. *(5 pts)* Find the formula for the velocity of the rocket cart at time $t \geq 0$.

b. *(5 pts)* Find the total distance traveled by the cart between $t = 0$ and $t = 3$.

Problem 7. *(10 points)* A small helium balloon is rising at the rate of 8 ft/sec, a horizontal distance of 12 feet from a 20 ft. lamppost. At what rate is the shadow of the balloon moving along the ground when the balloon is 5 feet above the ground?



Problem 8. *(10 points total)* There are two points on the ellipse

$$5x^2 - 6xy + 5y^2 = 4$$

with the x-coordinate equal to 1. Find where the tangent lines to the ellipse, at these two points, intersect.