Loyola University Chicago
Math 161, Fall 2010

Midterm 1

Name (print): _____ **Signature:** _____

Please do not start working until instructed to do so.

You have 75 minutes.

You must show your work to receive full credit.

No calculators.

You may use one double-sided 8.5 by 11 sheet of handwritten (by you) notes.

Problem 1. _____

Problem 2. _____

Problem 3. _____

Problem 4. _____

Problem 5. _____

Total. _____

Problem 1. *(40 points)* Find the following limits and derivatives. Put a box around your final answer.

a. *(5 points)* $\lim_{x \to 4} \dfrac{7x+2}{x^2-1}$

b. *(5 points)* $\lim_{y \to \infty} \dfrac{10y - 2y^2 + 5}{\sqrt{7 + y^2 + 9y^4}}$

c. *(5 points)* $\lim_{x \to -2^+} \sqrt{\dfrac{x^2+3x+2}{x^2+x-2}}$

d. *(5 points)* $\lim_{x \to 1^-} \dfrac{x^2+3x+2}{x^2+x-2}$

e. *(5 points)* $\lim_{z \to 3^-} \dfrac{2z-6}{|z-3|}$

f. *(5 points)* $\dfrac{d}{dx}\left(7x^5 - 11 + \ln(x) - \tan(x)\right)$

g. *(5 points)* $\dfrac{d}{dx}\left(\sqrt{5 + \cos(3^{4x})}\right)$

h. *(5 points)* $\dfrac{d}{dx}\left(\dfrac{a^x + b}{\sin x + x^c}\right)$ where a, b, and c are constants

Problem 2. *(10 points)* Use the definition of the derivative to find the derivative of $f(x) = \sqrt{5x^2 + 1}$.

Problem 3. *(15 points)* Consider the function

$$f(x) = \begin{cases} x^2 & \text{if} \quad x < 1 \\ 2x - 1 & \text{if} \quad 1 \leq x \leq 3 \\ 5 & \text{if} \quad x > 5 \end{cases}$$

a. *(5 points)* Is this function continuous at $x = 1$? Is this function continuous at $x = 3$. Justify your answer.

b. *(5 points)* Is this function differentiable at $x = 1$? Is this function differentiable at $x = 3$. Justify your answer.

c. *(5 points)* With $f(x)$ as above and $g(x)$ such that $g(1) = 2$ and $g'(1) = -4$, find $h'(1)$, where $h(x) = f(g(x))$.

Problem 4. *(15 points)* Consider the curve

$$x^2 - 4xy + y^4 = 0.$$

a. *(5 points)* Find the general formula for $\dfrac{dy}{dx}$, the slope of the curve, in terms of x and y.

b. *(5 points)* Find the equation of the line tangent to the curve at the point $\left(\frac{1+\sqrt{3}}{2}, 1\right)$.

c. *(5 points)* Find the point (or points) on the curve where the tangent line is horizontal.

Problem 5. *(10 points total)* When a bactericide was added to a nuntrient broth in which bacteria were growing, the bacterium population continued to grow for a while, but then stopped growing and began to decline. The size of the population, measured in thousands, at time $t > 0$ (hours) is

$$b(t) = te^{-\frac{t}{3}}.$$

a. *(5 points)* How fast is the bacterium population changing at time t?

b. *(5 points)* When does the population stop growing?

c. *(5 points)* What is the maximum size of the population?