Loyola University Chicago
Math 161-001, Fall 2010

Midterm 1 Sample

Name (print): ____________________ **Signature:** ________________

Please do not start working until instructed to do so.

You have 75 minutes.

You must show your work to receive full credit.

No calculators.

You may use one one-sided 8.5 by 11 sheet of handwritten (by you) notes.

Problem 1. ________

Problem 2. ________

Problem 3. ________

Problem 4. ________

Problem 5. ________

Problem 6. ________

Total. ________

Problem 1. *(30 points)* Find the following limits and derivatives. Put a $\boxed{\text{box around your final answer}}$.

a. *(5 points)* $\displaystyle\lim_{x\to 1} \frac{x^2+2x-3}{x-2+x^2}$

Solution:

$$\lim_{x\to 1} \frac{x^2+2x-3}{x-2+x^2} = \lim_{x\to 1} \frac{(x-1)(x+3)}{(x-1)(x+2)} = \lim_{x\to 1} \frac{x+3}{x+2} = \boxed{\frac{4}{3}}$$

b. *(5 points)* $\displaystyle\lim_{y\to -\infty} \frac{9y^4-5y^7}{2y^7-9y^4+55}$

Solution:

$$\lim_{y\to -\infty} \frac{9y^4-5y^7}{2y^7-9y^4+55} = \lim_{y\to -\infty} \frac{-5y^7+9y^4}{2y^7-9y^4+55}\frac{y^{-7}}{y^{-7}} = \lim_{y\to -\infty} \frac{-5+9y^{-3}}{2y-9y^{-3}+55y^{-7}} = \boxed{\frac{-5}{2}}$$

c. *(5 points)* $\displaystyle\lim_{x\to\infty} \sqrt{x^4+2x^3}-x^2$

Solution:

$$\lim_{x\to\infty} \sqrt{x^4+2x^3}-x^2\frac{\sqrt{x^4+2x^3}+x^2}{\sqrt{x^4+2x^3}+x^2} = \lim_{x\to\infty} \frac{2x^3}{\sqrt{x^4+2x^3}+x^2} = \lim_{x\to\infty} \frac{2x}{\sqrt{1+2x^{-1}}+1} = \boxed{\infty}$$

d. *(5 points)* $\displaystyle\frac{d}{dx}\left(e^{3x}\cos(5x)\right)$

Solution:

$$\frac{d}{dx}\left(e^{3x}\cos(5x)\right) = \boxed{3e^{3x}\cos(5x) - 5e^{3x}\sin(5x)}$$

e. *(5 points)* $\displaystyle\frac{d}{dz}\left(\frac{e^{az}}{z^3+b}\right)$

Solution:

$$\frac{d}{dz}\left(\frac{e^{az}}{z^3+b}\right) = \boxed{\frac{ae^{az}(z^3+b)-e^{az}3z^2}{(z^3+b)^2}}$$

f. *(5 points)* $\displaystyle\frac{d}{dx}\sqrt{x+\tan(5x)}$

Solution:

$$\frac{d}{dx}\sqrt{x+\tan(5x)} = \boxed{\frac{1}{2}(x+\tan(5x))^{-1/2}\left(1+\frac{5}{\cos^2(5x)}\right)}$$

Problem 2. *(10 points total)* Find horizontal and vertical asymptotes of $f(x) = \dfrac{1+4e^x}{5+2e^x}$.

Solution:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+4e^x}{5+2e^x} = \lim_{y\to\infty} \frac{1+4y}{5+2y} = \frac{4}{2} = 2$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1+4e^x}{5+2e^x} = \lim_{z\to 0} \frac{1+4z}{5+2z} = \frac{1}{5}$$

Thus, there are two horizontal asymptotes: $y = 2$ *and* $y = 1/5$.

There are no vertical asymptotes because $e^x > 0$ *for all* x*, so* $5 + 2e^x > 5$*, the denominator is never* 0*, and so the function is continuous.*

Problem 3. *(10 points total)* The half-life of phosphorus-32 is 14 days. There are 10 grams present initially. When will there be 1 gram remaining?

Solution: general formula for exponential decay is $Q(t) = Q(0)e^{-kt}$*. Half-life is* 14*, so* $\frac{1}{2} = e^{-k14}$, $\ln\left(\frac{1}{2}\right) = -14k$, $k = \frac{\ln 2}{14}$*. Then*

$$Q(t) = Q(0)e^{-\frac{\ln 2}{14}t} = 10\left(\frac{1}{2}\right)^{t/14}$$

Need to find out when $Q(t) = 1$*. So* $1 = 10e^{-\frac{\ln 2}{14}t}$, $\ln\frac{1}{10} = -\frac{\ln 2}{14}t$, $14\ln 10 = \ln 2t$*, and finally,*

$$t = \frac{14\ln 10}{\ln 2}$$

Problem 4. *(10 points)* Use the definition of the derivative to find the derivative of $f(x) = \dfrac{1}{1+3x}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{1+3(x+h)} - \frac{1}{1+3x}}{h} = \lim_{h\to 0} \frac{\frac{1}{1+3(x+h)} - \frac{1}{1+3x}}{h} \frac{(1+3(x+h))(1+3x)}{(1+3(x+h))(1+3x)} \\ &= \lim_{h\to 0} \frac{1+3x-(1+3x+3h)}{h(1+3(x+h))(1+3x)} = \lim_{h\to 0} \frac{-3h}{h(1+3(x+h))(1+3x)} \\ &= \lim_{h\to 0} \frac{-3}{(1+3(x+h))(1+3x)} = \frac{-3}{(1+3x)^2} \end{aligned}$$

Problem 5. *(10 points total)* Let f be defined by $f(x) = x|x|$. If f is differentiable at $x = 0$, find $f'(0)$. If f is not differentiable at $x = 0$, explain why. In either case, be sure to show all your work.

Solution: f is differentiable at $x = 0$ and $f'(0) = 0$. This is why: for $x \geq 0$, $|x| = x$, so $x|x| = x^2$, and

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h^2}{h} = \lim_{h\to 0^+} h = 0$$

while for $x \leq 0$, $|x| = -x$, so $x|x| = -x^2$, and

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{-h^2}{h} = \lim_{h\to 0^+} -h = 0$$

The left and the right limit exist, and are equal to each other, hence

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = 0$$

Problem 6. *(20 points)* A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. The rock reaches a height of $h = 160t - 16t^2$ ft after t seconds.

a. *(5 points)* How high does the rock go?

b. *(5 points)* What is the velocity and speed of the rock when it is 256 ft above the ground, on the way up? On the way down?

c. *(5 points)* What is the acceleration of the rock at any time t during its flight?

d. *(5 points)* When does the rock hit the ground again?

Solution: see Example 4 in Section 3.4 of the textbook.