

Name (print): \_\_\_\_\_ Signature: \_\_\_\_\_

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Please do not start working until instructed to do so.

You have 75 minutes.

You must show your work to receive full credit.

No calculators.

You may use one double-sided 8.5 by 11 sheet of handwritten (by you) notes.

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Problem 1. \_\_\_\_\_

Problem 2. \_\_\_\_\_

Problem 3. \_\_\_\_\_

Problem 4. \_\_\_\_\_

Problem 5. \_\_\_\_\_

**Total.** \_\_\_\_\_

**Problem 1.** (20 points) Find the following limits and integrals. Put a box around your final answer.

**a.** (4 points)  $\lim_{x \rightarrow 1} \frac{x^7 - 1}{x^3 - 1}$

**b.** (4 points)  $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$

**c.** (4 points)  $\lim_{t \rightarrow 0} \frac{t \sin t}{1 - \cos t}$

**d.** (4 points)  $\int \sqrt{x} - \frac{5}{x^3} dx$

**e.** (4 points)  $\int \frac{e^{3x} - 4x}{11} dx$

**Problem 2.** (10 points) Use Newton's method to approximate the solution to  $x^4 + x = 3$ . Pick the initial approximation  $x_0$  reasonably. Then find  $x_1$  and  $x_2$ . You do not need to simplify  $x_2$ .

**Problem 3.** (10 points) When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

**Problem 4.** (10 points) Let  $f(x) = xe^{-x^2}$ .

**a.** Determine all horizontal and all vertical asymptotes of the curve  $y = f(x)$ .

**b.** Find all critical points of  $f$ .

**c.** Find the intervals on which  $f$  is increasing and the intervals on which  $f$  is decreasing.

**d.** Find all values  $x$  where  $f''(x) = 0$ .

**Problem 5.** (10 points) A load must be suspended 6 meters below a high ceiling using cables attached to two supports that are 2 meters apart (see figure). How far below the ceiling (labeled  $x$ ) should the cables be joined to minimize the total length of cable used? What is the minimum amount of cable needed? (Leave your answers in **exact** form and make sure to justify your minimum in some way.)

