Loyola University Chicago **Sample Quiz 2**
Math 161, Section 001, Fall 2010

Name (print): ______________________ **Signature:** ______________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (6 pts) Find the following limits:

a. $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$

Solution: recall that $|x-2| = x-2$ *if* $x-2 \geq 0$*, so when* $x \geq 0$*, and* $|x-2| = -(x-2)$ *when* $x-2<0$*, so when* $x<2$*. Then*

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1$$

b. $\lim_{x\to 2^+} \frac{|x-2|}{x-2}$

Solution:

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$$

Problem 2. (4 pts) Find the equations of all vertical and all horizontal asymptotes of
$f(x) = \frac{3x^2+2}{x^2+3+4x}$

Solution: to find horizontal asymptote(s):

$$\lim_{x\to\infty} \frac{3x^2+2}{x^2+3+4x} = \lim_{x\to\infty} \frac{3+\frac{2}{x^2}}{1+\frac{3}{x^2}+\frac{4}{x^2}} = 3, \quad \textit{similarly} \lim_{x\to-\infty} \frac{3x^2+2}{x^2+3+4x} = 3$$

Hence, $y=3$ *is the only horizontal asymptote. To find vertical asymptote(s):*

$$x^2+3+4x=0,\ x^2+4x+3=0,\ (x+3)(x+1)=0$$

So $x=-1$ *and* $x=-3$ *may be vertical asymptotes. To check:*

$$\lim_{x\to-1^-} \frac{3x^2+2}{x^2+3+4x} = \lim_{x\to-1^-} \frac{3x^2+2}{(x+3)(x+1)} = \lim_{x\to-1^-} \frac{3x^2+2}{x+3} \lim_{x\to-1^-} \frac{1}{x+1} = \frac{5}{2} \lim_{x\to-1^-} \frac{1}{x+1}$$

and the last limit is $-\infty$*. Hence* $x=-1$ *is a vertical asymptote. Similar argument shows that* $x=-3$ *is also a vertical asymptote.*

Problem 3. (6 pts) Find the following limit. Your answer may depend on x.

$$\lim_{h\to 0}\frac{\sqrt{1+x^2+h}-\sqrt{1+x^2}}{h}$$

Solution:

$$\begin{aligned}\lim_{h\to 0}\frac{\sqrt{1+x^2+h}-\sqrt{1+x^2}}{h} &= \lim_{h\to 0}\frac{\sqrt{1+x^2+h}-\sqrt{1+x^2}}{h}\frac{\sqrt{1+x^2+h}+\sqrt{1+x^2}}{\sqrt{1+x^2+h}+\sqrt{1+x^2}}\\ &= \lim_{h\to 0} = \frac{1}{\sqrt{1+x^2+h}+\sqrt{1+x^2}}\\ &= \frac{1}{2\sqrt{1+x^2}}\end{aligned}$$

Problem 4. (5 pts) For what value or values of c is the following function continuous:

$$f(t)=\begin{cases} t^2-10 & \text{for} \quad t\le c\\ 4+5t & \text{for} \quad t>c\end{cases}$$

Solution: we need $\lim_{t\to c-} t^2-10 = c^2-10 = \lim_{t\to c^+} 4+5t$ *which simplifies to* $c^2-10=4+5c$. *Solve, get* $c=-2$ *or* $c=7$.

Problem 5. (6 pts) Find the following limit. Your answer may depend on a.

$$\lim_{x\to\infty}\frac{3ax^2+ax+2}{3x+1}-ax-4$$

Solution:

$$\lim_{x\to\infty}\frac{3ax^2+ax+2}{3x+1}-ax-4=\lim_{x\to\infty}\frac{3ax^2+ax+2-(3x+1)(ax+4)}{3x+1}=\lim_{x\to\infty}\frac{-12x-2}{3x+1}=-4$$