**Loyola University Chicago** **Quiz 1**
**Math 161, Section 001, Fall 2010**

**Name (print):** _____ **Signature:** _____

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

**Problem 1.** (10 pts) Find the following limits:

**a.** $\lim_{x\to 5} 11x + e^{x-4} + \dfrac{x}{x+1}$

*Solution: set* $x = 5$. *Get* $\lim_{x\to 5} 11x + e^{x-4} + \frac{x}{x+1} = 55 + e + \frac{5}{6}$.

**b.** $\lim_{x\to 3^+} \dfrac{x-5}{x-3}$

*Solution: when* $x \to 3^+$, $x-5 \to -2$, $x-3 \to 0$ *while* $x-3 > 0$, *so* $\lim_{x\to 3^+} \frac{x-5}{x-3} = -\infty$

**c.** $\lim_{x\to 0} \dfrac{\tan 4x}{x}$ *(You can use the fact that* $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$*)*

*Solution:*

$$\lim_{x\to 0} \frac{\tan 4x}{x} = \lim_{x\to 0} \frac{1}{\cos 4x}\frac{\sin 4x}{x} = \lim_{x\to 0} \frac{1}{\cos 4x} \lim_{x\to 0} \frac{\sin 4x}{x} = \frac{1}{1} \lim_{x\to 0} 4\frac{\sin 4x}{4x} = 4 \lim_{x\to 0} \frac{\sin 4x}{4x} = 4$$

**d.** $\lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7}$

*Solution:*

$$\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3} = \lim_{x\to 7} \frac{x+2-9}{(x-7)\sqrt{x+2}+3} = \lim_{x\to 7} \frac{1}{\sqrt{x+2}+3} = \frac{1}{6}$$

**e.** $\lim_{h\to 0} \dfrac{\frac{7}{x+h} - \frac{7}{x}}{h}$

*Solution:*

$$\lim_{h\to 0} \frac{\frac{7}{x+h} - \frac{7}{x}}{h} = \lim_{h\to 0} \frac{7x - 7(x+h)}{h(x+h)x} = \lim_{h\to 0} \frac{-7h)}{h(x+h)x} = \lim_{h\to 0} \frac{-7)}{(x+h)x} = -\frac{7}{x^2}$$

**Problem 2.** (5 pts) Find the horizontal and the vertical asymptote(s), if any exist, for the function

$$f(x) = \frac{x^2 + x - 12}{x^2 + 6x + 8}$$

*Solution:*

$$f(x) = \frac{x^2 + x - 12}{x^2 + 6x + 8} = \frac{(x+4)(x-3)}{(x+4)(x+2)}$$

*Candidates for vertical asymptotes are* $x = -4$ *and* $x = -2$. *Check:*

$$\lim_{x \to -4^{\pm}} f(x) = \lim_{x \to -4^{\pm}} \frac{(x-3)}{(x+2)} = \frac{-7}{-2} = \frac{7}{2}, \quad \lim_{x \to -2^{+}} f(x) = \lim_{x \to -2} \frac{(x-3)}{(x+2)} = -\infty$$

*So,* $x = -2$ *is a vertical asymptote, but* $x = -4$ *is not. Now, check*

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f(x) \frac{x^2 + x - 12}{x^2 + 6x + 8} = \lim_{x \to \infty} f(x) \frac{1 + 1/x - 12/x^2}{1 + 6/x + 8/x^2} = 1, \quad \lim_{x \to -\infty} f(x) = 1$$

*Hence* $y = 1$ *is the only horizontal asymptote.*

**Problem 3.** (5 pts) For what values of $a$ and $b$ is the following function continuous?

$$f(x) = \begin{cases} 3x + a & \text{if} \quad x < 2 \\ 7 & \text{if} \quad x = 2 \\ bx^2 - 9 & \text{if} \quad x > 2 \end{cases}$$

*Solution: need*

$$\lim_{x \to 2^{-}} f(x) = f(2) = \lim_{x \to 2^{+}} f(x)$$

$$3\,2 + a = 7 = b\,2^2 - 9$$

$$6 + a = 7, \quad 7 = 4b - 9$$

$$a = 1, \quad b = 4$$