Loyola University Chicago **Sample Quiz 3**
Math 161, Section 001, Fall 2010

Name (print): ______________________ **Signature:** ______________

You have 30 minutes. Show your work. Notes, calculators not allowed! Problems are on both pages.

Problem 1. (6 pts) Find the derivatives:

$\frac{d}{dx}\left(\ln(\tan(3x^7))\right)$

Solution:

$$\frac{d}{dx}\left(\ln(\tan(3x^7))\right) = \frac{1}{\tan(3x^7)}\frac{1}{\cos^2(3x^7)}21x^6$$

$\frac{d}{dz}\left((\arctan(7z))^4\right)$

Solution:

$$\frac{d}{dz}\left((\arctan(7z))^4\right) = 4\left(\arctan(7z)\right)^3\frac{1}{1+(7z)^2}7$$

Problem 2. (5 pts) Use linear approximation to approximate $\sqrt[3]{25}$.

Solution: Notice that $\sqrt{27} = 3$. *Let* $f(x) = \sqrt[3]{x}$. *Let* $a = 27$. *Then* $f(a) = 3$, $f'(x) = \frac{1}{3}x^{-2/3}$, $f'(a) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$. *The linear approximation of* $f(x)$ *at* $x = a$ *is then*

$$L_a(x) = f(a) + f'(a)(x-a) = 3 + \frac{1}{27}(x-27)$$

Plug in $x = 25$, *get*

$$3 + \frac{1}{27}(-2) = 2\frac{25}{27}$$

By the way: $2\frac{25}{27} \approx 2.9259$ *and the real value is* $\sqrt[3]{25} \approx 2.9240$.

Problem 3. (4 pts) Simplify $\tan(\arccos x)$ to an expression not involving trig functions.

Solution: Consider a right triangle with one acute angle α, hypotenuse 1, adjacent side x. Then $\cos\alpha = x$, so $\alpha = \arccos x$. Then the opposite side is $\sqrt{1-x^2}$, and so

$$\tan(\arccos x) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$$

Problem 4. (5 pts) Water is flowing at the rate of 50 $\mathrm{m}^3/\mathrm{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. How fast is the water level falling when the water is 5 m deep?

Solution: Know $\frac{dV}{dt} = -50$, need $\frac{dh}{dt}$, where $V = \frac{1}{3}\pi r^2 h$, and similar triangles suggest that $\frac{r}{h} = \frac{45}{6}$. Then $r = \frac{45}{6}h$, and

$$V = \frac{1}{3}\pi\left(\frac{45}{6}h\right)^2 h = \frac{45^2}{3\,6^2}\pi h^3$$

Then

$$\frac{dV}{dt} = \frac{45^2}{3\,6^2}\pi 3h^2\frac{dh}{dt}$$

We are interested in the moment when $h = 5$, so

$$-50 = \frac{45^2}{3\,6^2}\pi 3\,5^2\frac{dh}{dt}$$

Consequently

$$\frac{dh}{dt} = -\frac{50\,6^2}{\pi\,45^2\,5^2}$$