Loyola University Chicago
Math 161, Section 001, Fall 2010

Quiz 4

Name (print): _____ **Signature:** _____

You have 30 minutes. Show your work. Notes, calculators not allowed! Problems are on both pages.

Problem 1. (6 pts) Find the limits:

$$\lim_{x\to\infty} \frac{x^2-5x}{\ln(3x)+1}$$

Solution:

$$\lim_{x\to\infty} \frac{x^2-5x}{\ln(3x)+1} = \lim_{x\to\infty} \frac{2x-5}{\frac{1}{x}} = \lim_{x\to\infty} 2x^2-5x = \infty$$

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}$$

Solution:

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} = \lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \lim_{x\to 0} \frac{-\sin x}{6x} = -\frac{1}{6}$$

Problem 2. (5 pts) Find all local minima and all local maxima for $f(x) = x^7(x-2)^6$.

Solution:

$$f'(x) = 7x^6(x-2)^6 + x^7 6(x-2)^5 = x^6(x-2)^5(13x-14)$$

Critical points: 0, 14/13, 2. *Local maximum at* 14/13. *Local minimum at* 2. *Nothing at* 0.

Problem 3. (3 pts) Find the limit:

$$\lim_{x\to 1} \frac{x^3 - 3x + 2}{2x^3 - x^2 - 4x + 3}$$

Solution:

$$\lim_{x\to 1} \frac{x^3 - 3x + 2}{2x^3 - x^2 - 4x + 3} = \lim_{x\to 1} \frac{3x^2 - 3}{6x^2 - 2x - 4} = \lim_{x\to 1} \frac{6x}{12x - 2} = \frac{6}{10}$$

Problem 4. (6 pts) Jane is located 8 km out from the nearest point A along a straight shoreline in her sea kayak. Hunger strikes and she wants to make it home for lunch; see picture. Jane can paddle at 3 km/hr and walk at 6 km/hr. She wants to make it home as soon as possible. Where should she beach the kayak and how long will it take her to get home?

Solution: Let x be the distance from A to the point where kayak reaches shore. The total time of travel home is

$$T(x) = \frac{\sqrt{8^2 + x^2}}{3} + \frac{9 - x}{6}$$

One needs to minimize this over x in $[0, 9]$. One obtains the minimum at

$$x = \frac{8}{\sqrt{3}}.$$

The minimum time to get home is

$$\frac{\sqrt{8^2 + (\frac{8}{\sqrt{3}})^2}}{3} + \frac{9 - \frac{8}{\sqrt{3}}}{6}$$