Loyola University Chicago
Math 161, Section 001, Fall 2010

Quiz 5 Sample

Name (print): ______________________ **Signature:** ______________

You have 30 minutes. Show your work. Notes, calculators not allowed! Problems are on both pages.

Problem 1. (4 pts) Let $f(x) = \begin{cases} x & \text{if} \quad x < 3 \\ 3 & \text{if} \quad x \geq 3 \end{cases}$. Find the average value of $f(x)$ on $[1, 5]$.

Solution:

$$A.V. = \frac{1}{5-1}\int_1^5 f(x)\,dx = \frac{1}{5-1}\left(\int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx\right) = \frac{1}{4}(4+6) = \frac{10}{4}$$

One can use geometry/area to find $\int_1^3 f(x)\,dx$ *and* $\int_3^5 f(x)\,dx$.

Problem 2. (3 pts) Evaluate

$$\int_{-4}^{4} 7 - \sqrt{16 - x^2}\,dx$$

Solution:

$$\int_{-4}^{4} 7 - \sqrt{16-x^2}\,dx = \int_{-4}^{4} 7\,dx - \int_{-4}^{4} \sqrt{16-x^2}\,dx = 7 \cdot 8 - \frac{1}{2}\pi 4^2 = 56 - 8\pi$$

Problem 3. (4 pts) Evaluate

$$\int 3x\sqrt{7-3x^2}\,dx$$

Solution: use $u = 7 - 3x^2$*, then* $du = -6x\,dx$*, and*

$$\int 3x\sqrt{7-3x^2}\,dx = \int 3x\sqrt{u}\frac{du}{-6x} = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\frac{2}{3}u^{\frac{3}{2}} + c = -\frac{1}{2}\frac{2}{3}(7-3x^2)^{\frac{3}{2}} + c = -\frac{1}{3}(7-3x^2)^{\frac{3}{2}} + c$$

Problem 4. (4 pts) Consider the integral $\int_1^3 e^x - x\,dx$.

- Which sum is an underestimate of the integral? *(Circle the correct answer.)*
  - The left-endpoint Riemann Sum
  - Right-endpoint Riemann sum
  - Neither
- Which sum is an overestimate of the integral? *(Circle the correct answer.)*
  - The left-endpoint Riemann Sum
  - Right-endpoint Riemann sum
  - Neither

(Very briefly) justify your answers!

Problem 5. (4 pts) If $\int_{-2}^{7} f(x)\,dx = 12$, $\int_{-2}^{0} f(x)\,dx = 20$, and $\int_5^7 f(x)\,dx = 4$, find

$$\int_0^5 3f(x) - x\,dx.$$

Solution: $\int_{-2}^{7} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx$ *and consequently*

$$\int_0^5 f(x)\,dx = 12 - 20 - 4 = -12.$$

Then

$$\int_0^5 3f(x) - x\,dx = 3\int_0^5 f(x)\,dx - \int_0^5 x\,dx = 3(-12) - \frac{1}{2}5 \cdot 5 = -36 - 12.5 = -48.5$$

Problem 6. (3 pts) Write the right-endpoint Riemann Sum for the function $f(x) = \ln(x)$ on the interval $[2, 4]$ with $n = 5$ subintervals of even length. *Do not evaluate the sum.*

Solution: $\Delta x = \frac{4-2}{5} = .4$, $x_0 = 2$, $x_1 = 2 + .4 = 2.4$, $x_2 = 2.8$, $x_3 = 3.2$, $x_4 = 3.6$, $x_5 = 4$. *The Riemann Sum is*

$$\Delta x\,(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)) = .4\,(\ln 2.4 + \ln 2.8 + \ln 3.2 + \ln 3.6 + \ln 4)$$