**Loyola University Chicago** **Quiz 5**
**Math 161, Section 001, Fall 2010**

**Name (print):** ______________________ **Signature:** ______________

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You have 30 minutes. Show your work. Notes, calculators not allowed! Problems are on both pages.

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**Problem 1.** (3 pts) Find the indefinite integral:

$$\int \cos(2x) + 5x\,dx$$

*Solution:*

$$\int \cos(2x) + 5x\,dx = \frac{1}{2}\sin(2x) + \frac{5}{2}x^2 + c$$

**Problem 2.** (4 pts) Find the antiderivative $F(x)$ of the function $f(x) = \sqrt{x} - 6$ such that $F(1) = 5$.

*Solution:*

$$\int \sqrt{x} - 6\,dx = \int x^{1/2} - 6\,dx = \frac{2}{3}x^{3/2} - 6x + c$$

*so every antiderivative has the form* $F(x) = \frac{2}{3}x^{3/2} - 6x + c$*. We need* $F(1) = 5$*, so*

$$5 = \frac{2}{3}1^{3/2} - 6 \cdot 1 + c$$

*solve for* $c$*, get* $c = 5 + 6 - \frac{2}{3} = 10\frac{1}{3}$*. So the answer is*

$$F(x) = \frac{2}{3}x^{3/2} - 6x + 10\frac{1}{3}$$

**Problem 3.** (4 pts) Write down the right-endpoint Riemann sum for $f(x) = \sin x - x$ on the interval $[0, 1]$ with $n = 3$ subintervals. Is this Riemann sum an overestimate of $\int_0^1 f(x)\,dx$ or an underestimate of $\int_0^1 f(x)\,dx$? Explain.

*Solution:*

$$RS = \frac{1}{3}\left(\sin\frac{1}{3} - \frac{1}{3}\right) + \frac{1}{3}\left(\sin\frac{2}{3} - \frac{2}{3}\right) + \frac{1}{3}(\sin 1 - 1)$$

*The Riemann sum is an underestimate because the function* $f$ *is decreasing on* $[0, 1]$*: we have* $f'(x) = \cos x - 1 \leq 0$.

**Problem 4.** (4 pts) Find $\int_{-3}^{7} f(x)\,dx$ where $f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } x < 0 \\ x+3 & \text{if } x \geq 0 \end{cases}$.

*Solution: break up the integral into two separate ones, find the separate ones using geometry.*

$$\int_{-3}^{7} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{7} f(x)\,dx = \int_{-3}^{0} \sqrt{9-x^2}\,dx + \int_{0}^{7} x+3\,dx = \frac{1}{4}\pi 3^2 + \frac{1}{2}(3+10)7 = \frac{9}{4}\pi + \frac{91}{2}$$

**Problem 5.** (4 pts) Find $b$ so that the average value of $f(x) = x - 4$ on the interval $[1, b]$ is 3.

*Solution:*

$$\frac{1}{b-1}\int_{1}^{b} x - 4\,dx = 3, \quad \left(\frac{1}{2}x^2 - 4x\right)\Big|_1^b = 3(b-1), \quad \frac{1}{2}b^2 - 4b - \frac{1}{2} + 4 = 3b - 3$$

*Simplify the lst equation, get*

$$\frac{1}{2}b^2 - 7b + 6\frac{1}{2} = 0, \quad b^2 - 14b + 13 = 0, \quad (b-1)(b-13) = 0$$

*The solution we need is* $b = 13$.

**Problem 6.** (3 pts) Find the indefinite integral:

$$\int x(x-1)^{17}\,dx$$

*Solution: let* $u = x - 1$*, then* $du = dx$*,* $x = u + 1$*, and*

$$\int x(x-1)^{17}dx = \int (u+1)u^{17}\,du = \int u^{18} + u^{17}\,du = \frac{1}{19}u^{19} + \frac{1}{18}u^{18} + c = \frac{1}{19}(x-1)^{19} + \frac{1}{18}(x-1)^{18} + c$$