Loyola University Chicago
Math 201, Fall 2009

Final Exam

Name (print): _____ **Signature:** _____

Please do not start working until instructed to do so.

You have 2 hours.

You must show your work to receive full credit.

You may use one double-sided 8.5 by 11 sheet of handwritten (by you) notes.

Problem 1. _____

Problem 2. _____

Problem 3. _____

Problem 4. _____

Problem 5. _____

Problem 6. _____

Problem 7. _____

Problem 8. _____

Problem 9. _____

Total. _____

Problem 1. *(10 points total)* Let P, Q, and R be statements. Is (P AND Q) $\implies$ R equivalent to ($P \implies R$) OR ($Q \implies R$)?

Problem 2. *(8 points total)* Multiply $(102)_3$ and $(20112)_3$ in base 3.

Problem 3. *(14 points total)*

a. *(7 points)* Find all integers x such that $x^3 + 4x + 1 \equiv 0 \pmod{5}$.

b. *(7 points)* Find all nonnegative integers such that $x^3 + 4x + 1 \equiv 0 \pmod{5}$ and $3x \equiv 2 \pmod{7}$.

Problem 4. *(10 points total)* Suppose that $ac \equiv bc \pmod{m}$ and $\gcd(c, m) = 1$. Show that $a \equiv b \pmod{m}$.

Problem 5. *(10 points total)* Let $x_0 = 5$, $x_1 = 1$, and $x_n = -5x_{n-1} + 14x_{n-2}$ for $n = 2, 3, \ldots$. Show that $x_n = 4 \cdot 2^n + (-7)^n$ for all $n = 0, 1, 2, \ldots$.

Problem 6. *(16 points)*

a. *(6 points)* Find $\gcd(990, 714)$.

b. *(5 points)* Find all integers x and y such that $990x + 714y = 18$.

c. *(5 points)* Find all integers x and y such that $990x + 714y = 25$.

Problem 7. *(10 points total)* Prove that $2^{70} + 3^{70}$ is divisible by 13.

Problem 8. *(12 points total)* How many elements does each of the following sets have? *Frame your answer. No partial credit.*

a. *(4 points)* Nonnegative divisors of 3969.

b. *(4 points)* Nonnegative integers, greater than 100 and less than 660, that can be written with distinct digits.

c. *(4 points)* Ways of ordering the elements of the set $\{A, B, C, D, E, F, G, H, I, J\}$ in a line such that either A or B is first and G is last.

Problem 9. *(10 points)* Prove using mathematical induction that there is $n!$ permutations of a set with n elements. *Only partial credit will be given for proofs that do not use induction. Recall that a permutation of a set with n elements is, essentially, an arrangements of n elements in a line.*