Loyola University Chicago
Math 201, Section 001, Fall 2009

Solutions to Quiz 1

Name (print): _____ **Signature:** _____

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (8 pts) Use truth tables to show the following:

P AND (Q OR R) is equivalent to (P AND Q) OR (P AND R)

Problem 2. (6 pts) Consider the following statement:

$$\textit{If } x \geq 0 \textit{ and } y \geq 0 \textit{ then } xy \geq 0.$$

Do the following:

a. Write the contrapositive statement and determine whether it is true or not.

Let P be "$x \geq 0$ and $y \geq 0$", let Q be "$xy \geq 0$". The statement is $P \implies Q$. The contrapositive is $NOTQ \implies NOTP$, and so

$$\textit{If } NOT\ xy \geq 0 \textit{ then } NOT\ (x \geq 0 \textit{ and } y \geq 0).$$

This simplifies to

$$\textit{If } xy < 0 \textit{ then } x < 0 \textit{ or } y < 0.$$

This statement is true. One way to justify this is to say that the original statement was true.

b. Write the converse statement and determine whether it is true or not.

The converse to $P \implies Q$ is $Q \implies P$. So,

$$\textit{If } xy \geq 0 \textit{ then } x \geq 0 \textit{ and } y \geq 0.$$

This statement is false. For example, take $x = y = -1$.

Problem 3. (6 pts) Prove or disprove the following statement:

$$x \in \left\{y \in \mathbb{R} \,|\, y^2 - 3y \leq 0\right\} \implies 0 \leq x \leq 5.$$

The set $\left\{y \in \mathbb{R} \,|\, y^2 - 3y \leq 0\right\}$ is the set $\{y \in \mathbb{R} \,|\, y(y-3) \leq 0\}$ which is the interval $[0, 3]$. If $x \in [0, 3]$ then $x \geq 0$ and $x \geq 3$. If $x \leq 3$ then $x \leq 5$. So, $x \in [0, 3]$ implies $0 \leq x \leq 5$. The statement is true.