Loyola University Chicago **Quiz 1**
Math 201, Spring 2010

Name (print): ______________________ **Signature:** ______________________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (4 pts) Write a truth table for the following expression:

$$(P \ OR \ NOT \, Q) \implies P.$$

Problem 2. (6 pts) Let P mean *I bike to work*, let Q mean *It is icy*, let R mean *My wrist hurts*. Write the following sentences using P, Q, R and connectives or vice-versa:

I biked to work when it was icy and now my wrist hurts.

Solution: $(P \ AND \ Q) \implies R$

$(Q \ OR \ R) \implies NOT \ P$

Solution: When it is icy or my wrist hurts, I do not bike to work.

It is necessary that it be icy for me to bike to work.

Solution: $P \implies Q.$

Problem 3. (4 pts) Write the contrapositive and the converse of the statement below, and clearly say whether the contrapositive is true or false and the whether the converse is true or false:

$$\textit{If } xy < 0 \textit{ then } x < 0 \textit{ or } y < 0.$$

Contrapositive:

Solution: If $x \geq 0$ and $y \geq 0$ then $xy \geq 0$. TRUE.

Converse:

Solution: If $x < 0$ or $y < 0$ then $xy < 0$. FALSE. (Take both x and y negative.)

Problem 4. (6 pts) Prove that, for any sets A, B, and C,

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

Note: $x \in T \setminus S$ means that $x \in T$ AND $x \notin S$. In the proof, you can use the fact that $P\, or\, (Q\, and\, R) \iff (P\, or\, Q)\, and\, (P\, or\, R)$ and that $P\, and\, (Q\, or\, R) \iff (P\, and\, Q)\, or\, (P\, and\, R)$, as long as you clearly state which one you are using and where.

Solution: The equality of the sets is equivalent to the following equivalence:

$$x \in A \setminus (B \cap C) \iff x \in (A \setminus B) \cup (A \setminus C).$$

This can be rewritten as:

$$x \in A \textit{ AND NOT } x \in (B \cap C) \iff x \in (A \setminus B) \textit{ OR } x \in (A \setminus C)$$

and then as

$$x \in A \textit{ AND } (x \notin B \textit{ OR } x \notin C) \iff (x \in A \textit{ AND } x \notin B) \textit{ OR } (x \in A \textit{ AND } x \notin C).$$

Let P be $x \in a$, let Q be $x \notin B$, let R be $x \notin C$. Then the equivalence above turns to

$$P\, and\, (Q\, or\, R) \iff (P\, and\, Q)\, or\, (P\, and\, R)$$

which is true.