Loyola University Chicago
Math 201, Section 001, Fall 2009

Quiz 2 Solutions

Name (print): ______________________ **Signature:** ______________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (5 pts) Let a,b, and c be integers. Prove that if $c|a$ and $c|b$, then $c|ax+by$ for any integers x and y.

Solution: $c|a$ *means that there exists an integer* m *such that* $a = mc$. $c|b$ *means that there exists an integer* n *such that* $b = nc$. *For any integers* x, y, *we have* $ax + by = mcx + ncy = (mx + ny)c$. *Hence* $c|ax + by$.

Problem 2. (5 pts) Add and multiply $(1121)_3$ and $(201)_3$ together in base 3.

Answer:

$$(1121)_3 + (201)_3 = (2022)_3$$

$$(1121)_3 \times (201)_3 = (1010021)_3$$

Problem 3. (5 pts) Find all the integer solutions to the Diophantine equation $7x + 9y = 1$.

Answer:

$$x = 4 - 9n, \quad y = -3 + 7n, \quad \textit{where } n \textit{ is integer}$$

Problem 4. (5 pts) Let a, b be integers. Suppose that $d = \gcd(a, b) \neq 0$. Prove that $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

Answer: When $d = gcd(a, b)$, there exist integer x, y such that $ax + by = d$. When $d \neq 0$, this is equivalent to $\frac{a}{d}x + \frac{b}{d}y = 1$. Since $\frac{a}{d}$ and $\frac{b}{d}$ are integers, and since 1 is a nonnegative common divisor of $\frac{a}{d}$ and $\frac{b}{d}$, the equation $\frac{a}{d}x + \frac{b}{d}y = 1$ implies that 1 is the greatest common divisor of $\frac{a}{d}$ and $\frac{b}{d}$. (Theorem 2.24 says so!)