Loyola University Chicago
Math 201, Spring 2010

Quiz 2

Name (print): _____ **Signature:** _____

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (5 pts) Let a and b be positive integers. Prove that if $a|b$ and $b|a$ then $a = b$.

Solution: $a|b$ so there exists $x \in \mathbb{Z}$ such that $b = ax$. Similarly, since $b|a$, there exists $y \in \mathbb{Z}$ such that $a = by$. Then $b = ax = byx$. Since $b > 0$, $1 = xy$. Then $x = y = 1$ or $x = y = -1$. The second option is impossible since both a, b are positive. Then $x = y = 1$, which gives $a = b$.

Alternatively: since both a, b are positive, $a|b$ implies $a \leq b$. Similarly, $b|a$ implies $b \leq a$. Hence $a = b$.

Problem 5. (5 pts) Convert 5 and 24 to base 3, multiply them in base 3, then convert the product back to base 10.

Solution: $5 = 3 + 2 = (12)_3$. $24 = 2 * 9 + 2 * 3 = (220)_3$.

$$\begin{array}{cccccc} & & & 2 & 2 & 0 \\ \times & & & & 1 & 2 \\ & & 1 & 2 & 1 & 0 \\ + & & 2 & 2 & 0 & \\ & 1 & 1 & 1 & 1 & 0 \end{array}$$

$(11110)_3 = 81 + 27 + 9 + 3 + 0 = 120$.

Problem 3. (5 pts) Find the greatest common divisor of 6^{11} and 10^7.

Solution: $6^{11} = 2^{11}3^{11}$. $10^7 = 2^7 5^7$. *The common factors are the seven* 2*'s. So gcd is* 2^7.

Problem 4. (5 pts) Find all pairs of nonnegative integers x and y solving the Diophantine equation

$$126x - 75y = 6.$$

Answer: $x = 6 + 25k$, $y = 10 + 42k$, $k = 0, 1, 2, \ldots$.

Extra: (3 pts) For any integers a and b, show that $\gcd(a, b) = \gcd(a, a + b)$.
Extra: (3 pts) Prove or disprove: if a, b are integers and $\gcd(a, b) = d$ then $\gcd(a + b, a - b) = d$.