

Name (print): _____ Signature: _____

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (6 pts) Is the number below divisible by 2? By 3? By 4? By 5? By 6? By 9? Very briefly explain each answer.

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Problem 2. (4 pts) Recall that $a \equiv b \pmod{m}$ means that $m \mid (a - b)$. Prove the following properties:

- (a) $a \equiv b \pmod{m}$ implies $b \equiv a \pmod{m}$,
- (b) $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ implies $a \equiv c \pmod{m}$.

Problem 3. (4 pts) Find the last digit in the representation of 7^{451} in base 10 and in base 8.

Problem 4. (8 pts) True or false? If true, give a brief proof/explanation. If false, give a counterexample.

If $15 \nmid a$ and $15 \nmid b$ then $15 \nmid ab$.

If two distinct prime numbers p and q are such that $pq \mid a^2$ then $p \mid a$ and $q \mid a$.

If $13 \mid ab$ then $13 \mid a$ or $13 \mid b$.

If two prime numbers p and q are such that $pq \mid a^2$ then $p \mid a$ and $q \mid a$.