Loyola University Chicago
Math 201, Spring 2010

Sample Quiz 3

Name (print): ______________________ **Signature:** ______________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (5 pts) Find the prime number factorizations of 990, 2772, and of the least common multiple of 990 and 2772.

Solution:

$$990 = 2 \cdot 495 = 2 \cdot 3 \cdot 165 = 2 \cdot 3^2 \cdot 55 = 2 \cdot 3^2 \cdot 5 \cdot 11,$$
$$2772 = 2 \cdot 1386 = 2^2 \cdot 693 = 2^2 \cdot 3 \cdot 231 = 2^2 \cdot 3^2 \cdot 77 = 2^2 \cdot 3^2 \cdot 7 \cdot 11,$$
$$lcm(990, 2772) = 2^2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11.$$

Problem 2. (5 pts) Find the remainder when 3^{663} is divided by by 7.

Solution: modulo 7*, we have* $3 \equiv 3$, $3^2 \equiv 2$, $3^3 \equiv 6 \equiv -1$, *so*

$$3^{663} \equiv \left(3^3\right)^{221} \equiv (-1)^{221} \equiv -1 \equiv 6.$$

Hence, the remainder is 6*.*

Problem 3. (5 pts) Prove that $6^{n+1} + 5 \cdot 3^n - 1$ is divisible by 10 for every natural number n.

Solution: divisibility by 10 *is equivalent to divisibility by* 2 *and by* 5*. Deal with* 2 *first:* 6^{n+1} *is always divisible by* 2 *because* 6 *is.* 3^n *is always odd, so* $5 \cdot 3^n$ *is odd. Hence* $6^{n+1} + 5 \cdot 3^n - 1$ *is even, i.e., divisible by* 2*. Deal with* 5 *now:* $5 \cdot 3^n$ *is divisible by* 5*, so one only needs to show* $5 | 6^{n+1} - 1$*. But* $6 \equiv 1 \pmod 5$ *so* $6^{n+1} \equiv 1 \pmod 5$ *so* $6^{n+1} - 1 \equiv 0 \pmod 5$*, which means* $5 | 6^{n+1} - 1$*. Done!*

Alternative solution: one can do this using mathematical induction, but this technique is not yet legal in our class.

Problem 4. (5 pts) Suppose that $a \equiv b \pmod m$ and $c \equiv d \pmod m$. Prove the following two statements:

- $a + c \equiv b + d \pmod m$,
- $ac \equiv bd \pmod m$.

Solution: See lecture notes or Proposition 3.12 in the textbook.