Loyola University Chicago **Quiz 3**
Math 201, Spring 2010

Name (print): ______________________ **Signature:** ______________________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (6 pts) Is the number below divisible by 2? By 3? By 4? By 5? By 6? By 9? Very briefly explain each answer.

1020304050601020304050 60

Solution:
Divisible by 2 *because last digit divisible by* 2.
Divisible by 3 *because sum of digits is* 42 *and* $3|42$.
Divisible by 4 *because last two digits, i.e.,* 60, *divisible by* 4.
Divisible by 5 *because last digit is* 0.
Divisible by 6 *because divisible by both* 2 *and* 3.
Not divisible by 9 *because sum of digits is* 42 *and* $9 \nmid 42$.

Problem 2. (4 pts) Recall that $a \equiv b \pmod m$ means that $m|(a-b)$. Prove the following properties:

(a) $a \equiv b \pmod m$ implies $b \equiv a \pmod m$,

(b) $a \equiv b \pmod m$ and $b \equiv c \pmod m$ implies $a \equiv c \pmod m$.

Solution: see textbook or lecture notes.

Problem 3. (4 pts) Find the last digit in the representation of 7^{451} in base 10 and in base 8.

Solution: $7^4 \equiv 1 \pmod{10}$*, so*

$$7^{451} = 7^{448}7^3 = \left(7^4\right)^{112} 7^3 \equiv 1^{112}7^3 = 7^3 \equiv 3 \pmod{10}$$

so the last digit in base 10 *is* 3*.*

$7 \equiv -1 \pmod{8}$ *so*

$$7^{451} \equiv (-1)^{451} = -1 \equiv 7 \pmod{8}$$

so the last digit in base 8 *is* 7*.*

Problem 4. (8 pts) True or false? If true, give a brief proof/explanation. If false, give a counterexample.

If $15 \not| a$ and $15 \not| b$ then $15 \not| ab$.

Solution: False. Try $a = 3$, $b = 5$.

If two distinct prime numbers p and q are such that $pq|a^2$ then $p|a$ and $q|a$.

Solution: True. p *is a prime factor of* a^2*, so also* p^2 *is a factor of* a^2*.* q *is a <u>different</u> prime factor of* a^2*, so also* q^2 *is a factor of* a^2 *<u>different</u> from* p^2*. Then* p *is a factor of* a *and* q *is a <u>different</u> factor of* a*.*

If $13|ab$ then $13|a$ or $13|b$.

Solution: True. 13 *is prime, and since it is a prime factor of* ab*, it is a prime factor of either* a *or* b *(or both). Then* $13|a$ *or* $13|b$*.*

If two prime numbers p and q are such that $pq|a^2$ then $p|a$ and $q|a$.

Solution: True. If p *and* q *are distinct, see two questions back. If* $p = q$ *then* $p|a^2$ *implies* p *is a prime factor of* a^2 *so* p^2 *is a factor of* a^2 *so* p *is a factor of* a*. Since* $q = p$*,* q *is a factor of* a *as well.*