Loyola University Chicago
Math 201, Spring 2010

Quiz 5

Name (print): ______________________ **Signature:** ______________________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (5 pts) Solve the simultaneous congruences

$$\begin{aligned} 4x &\equiv 3 \pmod{7} \\ 7x &\equiv 47 \pmod{23} \end{aligned}$$

Problem 2. (4 pts) State the Little Fermat's Theorem and use it to show that $15^{110} - 1$ is divisible by 11.

Problem 3. (5 pts) Find the inverse of [17] in $\mathbb{Z}_{53}$.
For full credit, your answer should have the form $[m]$, *where* m *is an integer between* 0 *and* 52.

Problem 4. (6 pts) The Chinese Remainder Theorem says the following: If $\gcd(m_1, m_2) = 1$, then, for any $a_1, a_2 \in \mathbb{Z}$, the simultaneous congruences $x \equiv a_1 \pmod{m_1}$, $x \equiv a_2 \pmod{m_2}$ have a solution, and if $x = x_0$ is one solution than the complete solution is $x \equiv x_0 \pmod{m_1 m_2}$. Use this result to prove the following:

- If $\gcd(m_1, m_2) = 1$ then
$$x \equiv a \pmod{m_1 m_2} \iff \begin{cases} x \equiv a \pmod{m_1} \\ x \equiv a \pmod{m_2} \end{cases}$$