Loyola University Chicago
Math 201, Spring 2010

Sample Quiz 4

Name (print): ______________________ **Signature:** ______________________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (7 pts) Find all solutions to the conguence

$$x^2 \equiv 4x \pmod{77}.$$

Problem 2. (3 pts) Find all solutions to

$$15x \equiv 14 \pmod{39}.$$

Problem 3. (5 pts) Prove that $n^{91} \equiv n^7 \pmod{91}$ for all integers n. Is $n^{91} \equiv n \pmod{91}$ for all integers n?

Problem 4. (5 pts) The Linear Congruence Theorem says that $ax \equiv c \pmod{m}$ has a solution if and only if $\gcd(a, m)|c$, and if x_0 is one solution then $x \equiv x_0 \pmod{\frac{m}{\gcd(a,m)}}$ is the complete solution. Use this result to prove the following:

- If $\gcd(m_1, m_2) = 1$, then, for any $a_1, a_2 \in \mathbb{Z}$, the simultaneous congruences

$$x \equiv a_1 \pmod{m_1}, \quad x \equiv a_2 \pmod{m_2}$$

have a solution, and if $x = x_0$ is one solution than the complete solution is $x \equiv x_0 \pmod{m_1 m_2}$.